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The epistemology of modern finance

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Abstract: Modern finance is a social science where the complexity of mathematical models compares to that of physics. The aim of this paper is to provide a conceptual framework for the interpretation of mathematical models in finance, in order to determine the epistemological standards according to which financial theory must be assessed. The analysis enlightens the contrast between highly objective results and the radical uncertainty that governs the markets. The main contribution of the paper is to show that the reasons why finance models are relative and non-causal are deeply rooted in the nature of finance theory itself. An important consequence is that arbitrage-free model prices are reference prices and indicators of the economical features underlying mathematical models. As such, they can be used to structure and support final pricing and hedging decisions, but not to predict future market prices.

Keywords: epistemology, mathematics, modern finance, risk, uncertainty

Introduction

Modern finance [1] has undergone an amazing expansion over the past decades, which goes along with the use of increasingly sophisticated mathematical models. However, the usefulness of advanced mathematical methods in finance remains questionable, as epitomized by McGoun (2003):

> By the traditionally rigorous standards of the natural sciences, financial economics has been a failure. It simply cannot predict anything with equivalent accuracy or reliability. (McGoun, 2003, p. 432)

The recent financial crisis has reinforced these criticisms. These, however, presuppose that financial economics must be assessed as a natural science, although it is a social science. Concededly, it is a highly quantitative discipline, where mathematical complexity often compares to that of the natural sciences. Yet, it does not follow that financial models must be evaluated according to the standards of the natural sciences. How should mathematical models in finance then be interpreted? This is the question the paper addresses. The aim is to provide a conceptual framework for the interpretation of mathematical models in finance, in order to determine the epistemological standards for assessing the validity of financial theory.

Consider a European call option, i.e., a contract that gives the holder the right to buy the underlying asset by a certain date (the maturity) for a certain price (the strike). The payoff of the option – the potential positive difference, at maturity, between the underlying asset's price and the strike – is intrinsically uncertain until the end of the contract. Under certain assumptions, option pricing theory gives a unique price for the option before maturity, independently of the agents' risk aversion. Given the random character of the payoff, this is a remarkable achievement. Even if this theory has emerged after other well-known theories (capital asset pricing model (CAPM), portfolio theory), the straightforward way it deals with uncertainty makes it an essential piece of the financial edifice, and the cornerstone of asset pricing. This is why the paper's investigation of modern finance zeroes on option pricing theory in particular.

Owing to this, the goal is to provide a new philosophical interpretation of modern finance's results.

Regarding the literature, the paper refers to various considerations on the epistemology of modern finance, featured in the works of e.g. McGoun (1997, 2003), McGoun and Zielonka (2006), Macintosh (2003) or Orléan (2005). Here, however, the perspective is more focussed on the mathematical nature of financial models, through the analysis of seminal works in the field (Black and Scholes, 1973; Harrison and Kreps, 1979) in the light of important contributions in the philosophy of mathematics and natural sciences (Fréchet, 1955; Bernays, 1976; Friedman, 1974). The main contribution of the paper is to analyze the intrinsic validity of financial models from an epistemological point of view. The novelty is in showing that the reasons why finance models are relative and non-causal are deeply rooted in the nature of finance theory itself.

The paper is organised as follows. Section 2 briefly sketches the genesis of mathematical finance, where risk is used to model uncertainty and where Black-Scholes option pricing model is the paradigm for dealing with it. Section 3 deepens the comparison between modern finance and the natural sciences, enlightening the role of mathematical models. Section 4 analyzes the understanding provided by mathematics in finance. Section 5 examines the important issue of self-reference in finance, through the concepts of self-fulfilling prophecy and redundancy. Section 6 tackles the delicate question of the contrast between global probabilistic models in financial theory and individual agents' interest in particular prices in practice. Finally, Section 7 focuses on the relation between financial theory and the real world of investors. Section 8 concludes.

Mathematical finance and Black-Scholes model

The theoretical development of mathematical finance starts with the arbitrage theorem. It is known as the fundamental theorem of finance and "It can be traced to the invention by Arrow (1964) of the general equilibrium model of security markets" (Duffie, 1996, p. xiii). An arbitrage opportunity exists when it is possible to make a riskless profit higher than the riskless rate of return, by taking simultaneous positions in different assets. This notion is central in finance for it is used as an argument to determine the fair price of a financial asset: if there are no arbitrage opportunities, the price of a security is said to be at a fair level.

Actually, the absence of opportunities of arbitrage (AOA) argument rests upon the idea of a world where all risks are insurable, which opens the door to options, because these are instruments that can transform a potential risk into an insured risk. In 1973, Black and Scholes make what might be considered as the most fundamental single breakthrough of the last decades when they come up with their option pricing model (OPM), providing the mathematical foundations to price risk, based on the AOA assumption. Their findings launched the era of mathematical finance, which is still developing at an amazing pace today.

The Black-Scholes model assumes that trading occurs continuously and that the stock price at time t, S_t, satisfies the following stochastic differential equation:

$$\frac{\mathrm{d}S_t}{S_t} = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t\,,$$

where μ is the constant (expected) drift and σ the constant volatility of the stock price. W_t is a Brownian motion (or Wiener process) that models the random part of stock prices – the infinitesimal increment dW_t being understood in Itô's sense (Friedman, 1975). S_t is said to follow a geometric Brownian motion [2].

Consider the purchase of a European call option with maturity T on a stock that is worth S_0 at time zero, with an exercise price equal to K. Then, the payoff of the call at maturity is equal to max $(S_T - K, 0)$, that is $S_T - K$ if $S_T > K$ and zero otherwise.

Before Black and Scholes, all the attempts to price options included unobservable parameters such as, for example, the expected return on the underlying asset – which is closely related to the risk-attitude (hence to utility functions) of individual investors (see e.g. Sprenkle, 1961). Black and Scholes were the first to come up with a formula for the valuation of options independent of any subjective parameter. They showed the irrelevance of the subjective probabilities associated with each possible price movement compared to the range (the volatility) of possible movements in the underlying asset's price.

Their central argument is that the market is complete, i.e., that any contingent claim can be replicated by a portfolio of other assets. Then, under the AOA assumption, they show that it is possible to create a riskless portfolio with stocks, call options and riskless bonds (in adequate proportions), whose value will not depend on the future price of the stock but only on time and on the values of objective and observable [3] constants: the current price and volatility of the stock, the exercise price and maturity of the option, and the riskless interest rate. Based on this instantaneous replication principle, they provide a partial differential equation that the option price, $C(S_t, t)$, must satisfy, as well as a closed form solution:

$$C(S_t, t) = S_t N(d_1) - Ke^{r(t-T)} N(d_2),$$

$$d_1 = \frac{\ln(S_t / K) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}},$$

$$d_2 = \frac{\ln(S_t / K) + \left(r - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}},$$

where the function $N(.)$ denotes the cumulative probability distribution function of a standardized normal variable.

The model is path-breaking for it explicitly solves the long-lasting option pricing problem, dating back to Bachelier's (1900) work, and at once introduces continuous-time stochastic calculus in finance, which has revealed very fertile for solving various problems. Soon, however, it appeared that several hypotheses were too stringent with respect to real financial markets, such as, for example, constant volatility and Brownian motion for describing the risky part of the underlying asset, no transaction costs, complete markets, etc. Since then, many models have tried to incorporate these features.

Here, we discuss the epistemological interpretation of Black-Scholes model, but the analysis extends to financial models relying on the AOA hypothesis in general. All these models generate so-called arbitrage-free prices. Neftci (1996, p. 13) proposes four usual arguments underlying the use of arbitrage-free prices:

(1) They can help to determine prices for new financial products;
(2) They can be used as benchmarks for scenario analysis in risk management;
(3) They can provide the market price of an illiquid asset, i.e., the asset price as if it was traded in a perfect market;
(4) Their deviation from observed market prices can be interpreted as mispricing, calling for arbitrage profits.

In the sequel, we examine these interpretations from an epistemological point of view, in the aim of bringing out the philosophical foundations for using mathematical models in finance. We start with the following paradox: despite the mathematically elaborated character of the underlying theory, financial models' predictive power (in terms of real market prices) seems weaker than that of models in the natural sciences.

Finance versus the natural sciences

Financial models are undoubtedly as complicated as those belonging to physics. There are famous examples of intellectual debts owed by finance vis-à-vis natural sciences, from the formulas of the heat transfer equation of physics which initially provided the solution to Black-Scholes partial differential equation to the famous Brownian motion, originally used to describe the motion of a particle that is subject to a large number of small molecular shocks.

However, unlike the physicist working with constants and forces that are universal (energy conservation, gravitation, nuclear forces, etc.), the financial theorist deals with problems that are quite similar to the statistician's problems (Arrow, 1970, p. 8). Financial models face a double uncertainty related, on the one hand, to the probabilistic nature of the outcome and, on the other hand, to the adequateness of the underlying hypotheses.

According to the classical picture of natural sciences, these are, in contrast, more assured concerning the initial assumptions they make. Consider the following example. Aristotle's physics advocates a very intuitive explanation about the movement query: each thing tends towards its own natural place, i.e., stones naturally tend towards the earth and fire naturally tends towards the sky. Today, the law of gravitation and the chemical explanation for combustion explain these phenomena differently. However, the facts that stones are attracted by the earth and fire by the sky, or that there is something like gravitational forces and oxygen combustion, all lean on reliable stylized facts: that stones, when thrown, fall back to the ground and candles, when lit, burn. The paradigmatic example of the stable assumption is the movement of planets. Whether modeled by the Ptolemaic epicycles theory or by Kepler's laws, planets keep revolving. This stability goes along with the possibility to make multiple and controlled experiments.

In financial economics, there is no such thing as the movement of planets and no real stability. The phenomena that are modeled are subject to highly unpredictable variations. Hence, the adequateness of the underlying assumptions concerning reality always has a fundamental probabilistic tint. Contrary to the classical ideal of natural sciences, financial theory directly deals with radical uncertainty. As Knight (1921, part III, chapter VIII, § 8) puts it, "At the bottom of the uncertainty problem in economics is the forward looking character of the economic process itself". This holds for modern finance as well.

Concededly, contemporary natural sciences abundantly use statistical methods, for example, when modeling random movements of gas particles. From that point of view, physics models the average behavior of particles and finance models the average expectation of each individual actor, as reflected in prices. There is, however, a crucial difference between statistical physics and financial modeling: the role of *people's* expectations, which has a strong influence on prices. In this sense, financial economics deals with the "knowledge of human conduct" rather than with the "knowledge of the external world", or with "the truths of logic and mathematics" (Knight, 1940, p. 6).

Consequently, according to the rigorous standards of the natural sciences, predictability in finance is fragile. This comes from the inherent variability of financial phenomena, which implies that finance helps deciding in a radically uncertain environment compared to statistical physics.

In everyday life, uncertainty is assessed from a subjective point of view, based on everyone's personal experiences, believes and judgements. In finance, uncertainty is dealt with through the concept of risk, as follows: pure uncertainty is split into several possible (future) states of the world, and objective probabilities are attached with each of them. As opposed to uncertainty, risk presupposes that one can evaluate the possible outcomes of a random phenomenon and the associated objective probabilities. In that sense, risk is the concrete materialization of the vaguer concept of uncertainty.

Going back to Knight's (1921, part I, chapter I, § 29) classical distinction between "risk as a known chance" and "true uncertainty", this conception of risk was first formalized by Arrow (1964) in his work on the general equilibrium model of security markets. Later, this fundamental setting enabled Black and Scholes to find out their OPM. Their approach was path-breaking for they made the price of a contract with an uncertain outcome (an option written on a stock) independent of the investors' aversion to risk. Thereby, they conferred a solid and objective basis to modern finance, independently of any subjective utility function, i.e., independently of the behavior of investors in the face of risk.

Historically, before this theoretical revolution, the notions of risk and uncertainty were surrounded by clouds of vagueness. The option revolution has created a setting where it is possible to price a new kind of risk, which has a very particular feature: replicability. This assumption is central in order to provide an accurate price for risk. In other words, the new definition of risk implies in essence that when risk is replicable, it can be priced, sold and, as a consequence, hedged. To put it otherwise, the range of pure uncertainty has been restricted for there is a part of risk – replicable risk – that is now manageable. This has enabled finance to deal with risk, and thus (to some extent) uncertainty, in an unrecorded precise way.

However, this objectivity also has intrinsic limits: it does not say anything about *absolute* quantities. The real world is made of relations between different parameters and these parameters have absolute values. At the end of the day, what matters is the absolute value of a variable. When the variable at stake is risk, the objectivity attached with the price of it is only relative for if it were absolute, uncertainty would *de facto* disappear. This is why financial results should be interpreted as *relative* in the sense that they rely on prices (and risks) of other assets which are supposed to be known. This difference is best pictured by Cochrane (2001):

> "In *absolute pricing*, we price each asset by reference to its exposure to fundamental sources of macroeconomic risk. The CAPM [is the] paradigm of the absolute approach [...]. In *relative pricing*, [...] we ask what we can learn about an asset's value *given* the prices of some other assets. We do not ask where the prices of other assets came from, and we use as little information about fundamental risk factors as possible. Black-Scholes option pricing is the classic example of this approach. While limited in scope, this approach offers precision in many applications". (Cochrane, 2001, p. xiv, original italics)

The absolute/relative distinction is a subtle but crucial nuance deeply rooted in modern finance's mathematical nature. The issue is then to characterize the epistemological status of modern finance as a relative pricing theory, i.e., to define more accurately the way financial results ought to be interpreted *as such*, without fundamentally questioning the underlying hypotheses. In the next section, we analyze the understanding provided by mathematics in finance.

Knowledge and mathematics in finance

Simplify the world in order to model it

The success of mathematics in natural sciences is unchallenged. The rationale behind it is less consensual. There is, grossly speaking, one alternative: either mathematics describes (and perfectly matches) the true essence of the world on an *a-priori* basis, or it is the "science of idealized structures" (Bernays, 1976, p. 210), i.e., a very coherent conceptual tool that can be applied successfully to natural phenomena, provided that these were transformed into mathematical problems. The first conception is problematic mainly because it presumes a perfect correspondence between theory and practice. More realistically, the other conception implies a mathematization of the world, in the sense that the mathematical characteristics of the basic material must first be determined and then modeled. To build a consistent theory, one must therefore overcome the obstacles due to the complex nature of the matter, that is, simplify the world in order to model it.

Such simplification requires restrictions which are specific to each type of scientific knowledge. In physics, often deemed to be the paradigm of any possible knowledge, the predictive power of mathematical models is paramount. When astrophysicists model the movement of planets, they rely on the fact that these movements are stable and repetitive. Finance, in contrast, models uncertainty itself, by means of pricing models that incorporate risk, which makes the prediction requirement somewhat contradictory with its nature: highly uncertain movements are modelled, that give no guarantee to happen similarly in the future. Thus, prediction (and explanation) cannot be the same as in the natural sciences. Nevertheless, there is, in a certain sense, a very strong objectivity in finance, which relies on mathematical methods and basic assumptions that require a specific interpretation – different from that of natural sciences. This calls for a more detailed examination of quantitative methods in finance.

Beyond the causal mechanical approach to explanation

The use of arbitrage-free model prices for making arbitrage profits with respect to observed market prices (see Section 2) implies that financial models, with all their mathematical background, are normative and describe how the world *should* be. Oppositely, one could advocate that financial models are positive and only describe how the world actually *is*. Cochrane (2001) describes the alternative very clearly:

> Asset pricing theory shares the positive versus normative tension present in the rest of economics. Does it describe the way the world *does* work, or the way the world *should* work? We observe prices or returns of many assets. We can use the theory positively, to try to understand why prices or returns are what they are. If the world does not obey a model's predictions, we can decide that the model needs improvement. However, we can also decide that the *world* is wrong, that some assets are 'mis-priced' and present trading opportunities for the shrewd investor. (Cochrane, 2001, p. xiii)

Stated otherwise, does departure from theoretical prices raise opportunities of arbitrage or does it simply indicate the need for a new model? The answer depends on the systematic nature of the deviation in question. If divergence from reality is systematic, then the model is wrong; if not, there is an opportunity for arbitrage. Focussing on the second situation – where one considers real prices as wrong with respect to a certain theoretical norm – it is still unclear whether prices can *really* be abnormal for, in a sense, the supply and demand dynamics that determines the prices in practice could be regarded as *the* only norm. Therefore, in order to assess the understanding of mathematical models in finance, it is necessary to go beyond the positive/normative alternative.

McGoun (2003, p. 422) suggests the term "useful framework" as "an epistemologically meaningful alternative to the traditional categorizations 'positive' and 'normative'". His breakthrough is to go beyond the causal mechanical approach to explanation in finance. Indeed, financial models are essentially relative models, pricing assets with respect to one another rather than providing results in terms of absolute magnitude. From the causal-mechanical point of view, they are unclassifiable for they give very consistent relations between different parameters without inducing any causality at all. This is related to the absence of a stable underlying pattern, the latter being *the* condition for getting causal predictions in terms of absolute values.

As suggested by McGoun, Friedman's (1974) unification type of explanation is a way to circumvent the classical deductive-nomological model, providing the hint for a conclusive epistemology of finance:

> Scientific explanation [...] increases our understanding of the world by reducing the number of independent phenomena that we have to accept as ultimate or given. A world with fewer independent phenomena is, other things equal, more comprehensible than one with more. (Friedman, 1974, p. 15)

This approach is called the unification approach for it primarily rests on the internal consistency and the range of the explanation, rather than on its ability to explain the real world in a causal way.

Allowing a central role to the unification approach to explanation goes along with recognizing the importance of mathematics in finance as a way to increase the understanding of the world. Mathematical models in finance help enlighten precise relations between phenomena that were previously unrelated or deemed independent (consider the link between an option and the underlying stock price).

Yet mathematics is not only an abstract black box to be used as a problem solver tool. When properly interpreted, it can offer original insights into the structure of concrete problems. In their seminal paper, Harrison and Kreps (1979) extended the theory of contingent claim valuation along the lines developed by Black and Scholes, giving "a definitive conceptual structure to the whole theory of dynamic security prices" (Duffie, 1996, p. xiv). They showed the correspondence between the existence of a unique equivalent martingale measure and the AOA assumption in complete markets. This abstract mathematical result has a deep economical signification: it shows that when every contingent claim can be replicated, i.e., when the market is complete, calculating arbitrage-free prices reduces to evaluating payoffs' expectations under a particular probability measure. This measure is the probability under which the underlying asset's drift is the riskless interest rate. Therefore, in complete markets, arbitrage-free pricing is equivalent to assuming that investors are risk-neutral.

Risk-neutrality is a notion that is closely related to the Black-Scholes OPM because the investors' risk preferences do not appear in the Black-Scholes partial differential equation, which makes the relation between the price of an option and the underlying asset's price independent of the investors' attitude towards risk. This was first formalized by Cox and Ross (1976). The critical insight is that risk-neutral valuation is equivalent (in complete markets) to the AOA, which is the fundamental principle of finance. This correspondence between a purely mathematical theorem (a formal equivalence result) and an economical feature (risk-neutral valuation) has revealed very fruitful – even beyond the field of option pricing.

Accordingly, mathematical results in finance may provide valuable insights that would have been very difficult to discover otherwise and that have a deep economical signification.

The absence of straightforward empirical confirmation or, as in Frankfurter and McGoun (2001, p. 407), the statement that the current paradigm needs "no empirical proof" does not mean that it is irreplaceable. On the contrary, it implies that the key criterion for the advent of a new theory is not the empirical support as such, but whether there is a reduction in the number of phenomena we have to accept as ultimate, and whether the theory entails logical (non necessarily causal) implications which unearth new economical features of the world. Of course, empirical confirmation is important but it cannot be the only decisive factor in evaluating modern finance, since the relation between financial theory and practice is not as direct as in other sciences. Indeed, modern finance enlightens the framework of reality in a mathematically coherent way, but without an unfailing predictive power because it actually prices risk in a framework that does neither rely on causality nor enable deductive-nomological predictions. Consequently, even when the underlying assumptions are respected, the validity of pricing theories essentially differs from the validity of physical laws, as described, for example, by the heat transfer equation.

By contrast, natural sciences thrive on existing phenomena and model them in a way that enables to predict them accurately. In fact, this type of prediction is misnamed for it only predicts something that already exists. Finance, on the contrary, cannot predict in the same way as natural sciences, given the absence of an underlying recurring structure. Therefore, financial results should not be considered as what market prices ought to be – this would imply that financial markets are regular enough to be actually predicted, which is not the case as shown, for example, by the recent financial crisis.

This tension between grounded quantitative methods and the lack of a stable underlying pattern calls for a specific epistemological interpretation, quite different from what mathematical research methods imply that it is.

Self-reference in finance

A self-fulfilling prophecy?

In characterizing the meaning of financial theory and its relation to the real world, the notion of self-fulfilling prophecy deserves some attention. In economic theory, it has been largely recognized that predictions on future prices affect current prices. The financial world offers a first-choice application area of this phenomenon which can cause, e.g., excess volatility, speculative bubbles, and self-fulfilling-prophecies (Adam and Szafarz, 1993; Brunnermeier, 2001; Siegel, 2003). These rely on a picture of reality that is neither entirely theoretical nor completely empirical, which very often leads to a realistic description of how and why an event actually happened. The fact that the theoretical models developed by academics are used by traders provides a perfect illustration of how finance creates in reality what it posited in theory.

Kuhn (1962) pioneered the philosophy of (natural) science by showing that, in scientific revolutions, a pertinent description of reality intertwines with the adoption of the associated vision of the world by a certain number of scientists. Each scientific revolution is thus associated with new social norms, institutional design and language. Similarly, the financial literature offers a certain consensus to admit that the option revolution did not occur exclusively on the basis of Black-Scholes OPM (Ferraro *et al.,* 2005). Other ingredients of the revolution were the creation of an official market for derivative securities (the Chicago Board Options

Exchange) at the same period and the invention of hand-held electronic calculators that were able to perform Black-Scholes calculations in a few seconds.

As stated by MacKenzie and Millo (2003, p. 107), "Option pricing theory – a 'crown jewel' of neoclassical economics – succeeded empirically not because it discovered pre-existing price patterns but because markets changed in ways that made its assumptions more accurate, and because the theory was used in arbitrage". Yet, beyond the self-fulfilling prophecy, they also emphasize the cultural and sociological aspects of the option revolution. In the same way, Orléan (2005) calls attention to the fact that the future representation is not an objective natural fact, but rather the result of a collective belief process, socially and historically constructed, shared by investors.

However, these considerations do not impact the scientific characteristics of the revolution in question; they only emphasize that asset pricing theory is intrinsically *relative*. What is more puzzling concerning the option revolution, as part of a social science, is that its high degree of (mathematical) objectivity seems to rely on internal consistency only. This requires shedding light on the role of redundancy in finance.

Redundancy and replication

Redundancy is ubiquitous in finance, leading to a very consistent theoretical body, but also to several problems of interpretation in terms of practical implementation. Concerning option pricing, it is clear that the existence of a replicating (or hedging) portfolio is essential. Given the dramatic success of contingent claim theory, this characteristic has "contaminated" all the field of asset pricing. In fact, the principle of pricing itself can be seen as leaning on redundancy for the price of an asset, S_t, must obey (Neftci, 1996, p. 281):

$$S_t = E_t\left[\frac{S_{t+1}}{1+R_t}\right].$$

That is, a simple asset's price is equal to the expectation of its future payoff, discounted with respect to its yield up to next period, R_t, which is a redundant definition for the yield is precisely the ratio between the future payoff and the asset price. A proper use of the Girsanov theorem makes it possible to find such a probability distribution that the asset's price becomes a martingale with respect to its future payoff discounted at the riskless rate. If there are no opportunities of arbitrage, this probability is a risk-neutral probability. If the AOA condition is respected, the redundant expression above can become a powerful, yet very simple pricing tool.

Interestingly, even the most trivial financial notion – an asset's price – relies on redundancy. Every asset is priced *by comparison* with a certain (expected) payoff, which shows that redundancy is the core of pricing. This is because finance relies on the following fundamental principle: "[…] In equilibrium, packages of financial claims which are, in essence, equivalent must command the same price" (Cox and Ross, 1976, p. 145, referring to Modigliani and Miller's (1958) major insight). This is also the primitive idea underlying the arbitrage theorem.

But can two assets be *really* equivalent? Even more, can their risks be *really* equivalent? Even when all the required hypotheses are fulfilled, it is ambiguous to affirm that an investor is indifferent between two redundant, yet still different assets. A good and its price in cash are different, even if a perfect market offer instantaneous switch from one to another. Moreover, trade instantaneity is impossible for at least two other variables distinguish the concept of value: time and risk.

By definition, two concrete things are never identical, otherwise there would only be a single one. It is enough to increase the level of precision in the observation to convince oneself that two different objects are never entirely equivalent. If the objects are material things, they never coincide totally, as far as time and space are concerned. If they are really distinct from one another, it follows that they differ at least considering the space they fill. And if they are not different, then there is only one object with no guarantee that it will remain identical as time passes by. These considerations originate in the logician's quarrel about the foundations of mathematics, with respect to the validity of expressions such as $a = a$ or $a = b$ and their role in the building of knowledge (Frege, 1892; Bernays, 1976). They also apply to financial securities and money. Strictly speaking, the perfect replication of an asset is possible only theoretically. As a consequence, there is a mismatch between the mathematically consistent replication principle (underlying pricing models) and the real replication of a single individual object, whatever it may be[4].

The nature of financial assets, i.e., mostly cash-flows and dematerialized assets, makes the identity principle in finance rather look like $a = b + c$. In that sense, replication is maybe more coherent in finance than anywhere else. Yet, strictly speaking, perfect replication is empirically impossible. Regarding the way financial theory relates to practice, this underlines the importance of keeping into account the intrinsic risk (e.g. liquidity risk) present in formulas based on replication. Similarly, the fact that $a = a$ is relative with respect to time recalls that models with calibrated parameters never guarantee the future value of an asset. Hence the mathematical approach in finance should go with the awareness that various non-negligible risks are implicitly present in financial models, because the basic financial atom is uncertainty.

A probabilistic interpretation

The relation between financial theory and practice is ambiguous for investors are primarily interested in specific prices, whereas "a probability-based reasoning always has a global, non individual shape" (Fréchet, 1955, p. 130, our translation). Indeed, the interpretation of financial models is restricted to a global level that does not entail any particular predictive power – despite the fact that very precise prices are provided – because finance primarily deals with future uncertainty. In fact, "The kind of understanding provided by science is global rather than local" (Friedman, 1974, p.18). Concerning finance, this is because the theory builds upon probability distributions, not individual prices paths. As Friedman (1974, p. 8) puts it, "Only particular events occur at definite times, and can thus actually be expected before their occurrence". Given the absence of a stable recurring pattern in finance, there are no definite times and hence no real predictions. Consequently, one should drop the requirement to predict particular events precisely when examining the validity of modern finance. Strictly speaking, one should abandon Popper's falsification methodology in finance, which is more adapted to the natural sciences.

Yet, reducing risk to a mathematical probability causes several problems in terms of interpretation. The ubiquity of the Gaussian related assumptions could be questioned, for example. Though, a more comprehensive theory of risk could favourably replace it, as is the case with Lévy models for example. Hence, the whole mathematical finance should not be disqualified on the ground that "the persistence of it in the face of its lack of success [is] an act of faith" (McGoun, 1995, p. 530). This theory is doing well in capturing risk, and even if radical uncertainty subsists, its range has been curbed. Moreover, risk is always modeled globally which means that considering financial results as local rather than global annihilates modern finance's validity, for its results are then straightforwardly applied to individual situations – which should not be the case *directly*. In this respect, modern finance's empirical success is, if not total, at least substantial.

The probabilistic interpretation is thus a strong argument in favour of the scientific nature of finance. However, it implies severe restrictions as regards the validity of financial theories at an individual's level. Financial models describe real markets consistently at an aggregate level. It is less obvious, however, that this validity also applies to particular situations. In finance, deducing from a general principle – which is consistent – the behavior of a particular actor is ambiguous in the sense that financial results sometimes considerably differ from what individual investors are actually willing to endorse.

The relation to the real world

Schematically, modern finance can be seen as a field composed of three different layers. At the bottom, the first one comprehends all the empirical functioning of financial life, where the laws of supply and demand are at work, one market participant searching to match the other in order to come up with a unique price. It is, in brief, the real world. The second layer is the market level. It relies on some (generally agreed) assumptions that enable it to function independently of all the particular vicissitudes of the first layer. There, prices are taken for granted and hypotheses are made concerning the completeness of markets in terms of liquidity, information, frictions, arbitrage, efficiency, etc. The third layer concerns any retransformation based on the prerequisites guaranteed by the second layer. At this level, all the modern theory of finance has been developed, relying on second-level assumptions such as the AOA.

This three-layer model epitomizes the problem in a very explicit way: the separation between the first and the two other levels is thicker than expected, which is another way to say that financial results are not straightforwardly deductible at an individual's level. As pointed out before, the crucial difference between finance and economics is that finance is a relative pricing theory: it does not elaborate beyond asset's prices. Economics, on the contrary, does not take prices for granted but tries to relate them to fundamental macroeconomic factors. Oppositely, in modern finance, the parting between the first and the second layer is hermetic, making finance's high degree of precision possible. This separation calls for the abandon of any eschatological ideal in financial theory, even more than in natural sciences.

From that perspective, the positive and normative interpretations of modern finance both are untenable (see also McGoun and Zielonka, 2006). The ambiguity of finance stems from the balance between precise objectivity and radical uncertainty. This ambivalence is also reflected in the three-layer model: mathematical objectivity is based on level two and developed at level three; the human facet of the problem consists of the endogenous link with level one – the world of investors.

The quest for finance's epistemological status can be reduced to assessing to which extent results from the level 2 (and 3) can be applied to level 1. This passage is delicate: theoretical generalizations, as coherent as they may appear, are not easily applied to individual actors of the economy for their personal interests may lead them to act differently from predicted, and to be right in acting so, with respect to their own interests. Even if certain real life markets are very close to ideal perfect markets, the personal history of investors makes it difficult to assert that model based calculations provide the right price for each of them, for nobody is perfectly rational. The point, however, is that everybody is sensitive to a rational argument.

In this view, there is effectively a correspondence between the structures underlying financial theories and those governing real financial markets, but it is not of a causal-mechanical nature. On the contrary, this correspondence is of a judgemental nature for the goal of modern finance is to provide a structure for deciding in a context of uncertainty – thanks to economically consistent financial models. From an epistemological point of view, the objectivity of modern finance is crucial for it supports and enhances economical and judgmental analysis. Importantly, this implies finding a certain balance between, on the one hand, mathematical complexity and, on the other hand, the transparency of both the model and its underlying hypotheses, without which financial theory becomes useless as a decision tool.

Before concluding, we come back to the standard utilizations of arbitrage-free model prices (see Section 2). Model prices can help to determine the prices of new financial products or the prices of illiquid assets, but they cannot be taken as *market* prices; instead, they must be considered as *reference* prices, because the relation between model and market prices is not deductive-nomological. The use of such prices as benchmarks for scenario analysis in risk management is better suited regarding the nature of financial models. Indeed, the non-causal character of arbitrage-free prices pleads for using them as indicators of the economical features underlying financial models. For example, in the context of market uncertainty, comparing prices of several models that cope with risk differently may reveal the risk sensitivity of the priced asset, which can help to structure the final pricing and hedging decision. This approach is in line with the nature of financial models. Moreover, it naturally extends to the case of incomplete markets, i.e., when risk cannot be completely hedged, because the non-uniqueness of prices in these markets forces to consider model prices as supports of risk analysis, instead of exact predictions of the (unique) future market price.

Conclusion

Mathematical models in finance must be interpreted in the unification sense, i.e., they help reduce the number of phenomena that were previously independent, and they offer original insights into the structure of concrete economical problems. Also, they may change the usual perspective on practical issues and, as in the case of option pricing and risk-neutral valuation, provide economically consistent tools that would have been very difficult to figure out otherwise.

Yet, quantitative financial models are fundamentally relative and non-causal, due to several reasons. First, abstract mathematical models in finance are probabilistic and have a global shape incompatible with the prediction of particular events. Second, financial models rely on past values of assets (through the calibration of models' parameters) and must therefore be considered only very carefully as estimations of future market prices. Third, replication is the keystone of financial theory but it is only an idealization; therefore, it is necessary to keep into account the intrinsic risk (e.g. liquidity risk) present in formulas based on replication. Fourth, the correspondence between hypotheses and reality is less direct in finance than in natural sciences for the underlying financial phenomena are highly variable. Consequently, financial models make sense only if the underlying hypotheses have a solid economical signification and if they are transparent enough for supporting and structuring consistent judgmental analysis. As such, model prices can be used to structure final pricing and hedging decisions, but not to predict future market prices.

Given the stochastic nature of almost all financial models, further research should investigate the role of the mathematical concept of probability in finance, its interpretation, and its relation to the real financial world.

Endnotes

[1] In the sequel, we use the terms finance, modern finance, modern finance theory, financial theory or financial economics equivalently.

[2] In their original paper, Black and Scholes make some further assumptions, which can be summarized as follows: the risk-free rate r is constant; no dividends are paid by the stock; there are no transaction costs; neither borrowing nor short-selling are restricted; all securities are perfectly divisible, i.e., it is possible to buy any fraction of a share.

[3] Except for the volatility parameter, which is not directly observable on the markets. In practice, it is inferred from past observations (see Section 5).

[4] Some authors (McGoun, 1997, or Macintosh, 2003) argue that contemporary finance is intrinsically self-referencing and should consequently adopt paradigms from literary theory, semiotics, linguistics, and semiology rather than continue to rely on economics-based theory. We argue that it is unrealistic to advocate such a paradigm shift. Rather, these philosophical paradigms (or at least the intuitions behind them) should serve to change our perspective on financial theory, by emphasizing that the level at which it touches practice is different from what is commonly thought.

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