More than a sum of its parts:
A Keynesian epistemology of statistics

Nicholas Werle

Abstract: The major theoretical insight of Keynes' *General Theory* is that aggregate quantities describing the state of an economy as a whole are irreducible to arithmetic summations of individual decisions. This breaks with the logic of classical political economy and establishes macroeconomics as the study of economy-wide dynamics, logically independent from any underlying theory of individual rationality. However, Keynes does have a theory of individual psychology that links expectations back up to aggregate quantities with robust statistical methods, which account for the fundamental uncertainty one faces in predicting the future. By comparing the theoretical structure of macroeconomics to that of thermodynamics and statistical mechanics, this essay proposes a novel reading of Keynes' epistemology of statistical laws. On this view, statistical methods allow theoreticians to connect the mechanics of vast numbers of micro-scale entities to a macro-scale dynamics, even in the absence of a fully determinate causal story. Keynes' belief that organic wholes emerge from the interactions of complex systems is a product of his early work on the development of statistical mechanics from kinetic theory. In light of this epistemological foundation, this essay shows how the neoclassical idea of supplying macroeconomics with microfoundations is inherently contradictory.

Keywords: Keynesian macroeconomics, probability, uncertainty, physics, econometrics

Introduction

Classical political economy starts and ends with the liberal individual. In this story, when the rational *homo economicus* meets others of his ilk, his natural

inclination to "truck, barter, and trade" compels him to engage in mutually beneficial exchange. Classical theory sees this moment as the organic birth of the market. Smith, Ricardo, *et al.* begin their theoretical work by analyzing how strictly rational agents navigate this exchange. This theory of individual decision-making bears the load of the entire classical edifice, which takes a market economy to be no more than the sum of these decentralized decisions. John Maynard Keynes, however, sees a fallacy of composition in this assumption. In his *General Theory of Employment, Interest, and Money*, Keynes argues that the economy as a whole has its own organic existence, irreducible to the individual agents making decisions within it. Thus, he inaugurates macroeconomics as the study of the whole economy, a discipline properly free from the classical microfoundations.

Rather than analyzing the implications of rational choices and markets for individual goods, macroeconomics works on aggregate quantities such as employment, national income, and effective demand. Keynes demonstrates how these data have an organic dynamical interaction that does not supervene on any theory of individual decision-making. Furthermore, they reflect the overall health of the economy in which people actually live. Despite the fact that it is irreducible to the analysis of rational choice, macroeconomics does have a fully developed theory of the individual psychology. Indeed, the way Keynes' *General Theory* describes decision-making is far more faithful to subjective experience than the idealized rational agent depicted in mathematical neoclassical theory. The Keynesian individual lives, works, and invests in a fundamentally uncertain world, and the nature of his expectations reflects that indeterminacy. In making predictions about the future, he relies on his recent experience, not double integrals. His decisions depend more on confidence than on rationality.

This essay proposes a novel reading of Keynes' *General Theory*, tracing its intellectual roots back to nineteenth century physics in order to understand the complex relationship between individuals and macroeconomics' aggregate quantities. During the 1800s, statistical mechanics and thermodynamics emerged as complementary theories of matter on different scales. While the former uses robust statistical methods to characterize the interactions of vast numbers of particles and the latter describes the behavior of matter in bulk, the two theories are irreducible to each other in just the same way Keynes' macroeconomics is irreducible to his account of individual psychology. This reading is buoyed by Keynes' earlier *Treatise on Probability*, which develops a subtle epistemology of

probabilistic and statistical laws, working on the exact physical theories in question. The *Treatise* shows how statistical laws link the particulate-level mechanics of a complex system to an emergent dynamics, logically independent from any theory of those fine-grained interactions. As a result, neoclassical attempts to produce a macroeconomics with microfoundations are inherently contradictory.

Of course, neoclassical econometricians are much enamored of performing linear regressions and risk analysis, but this reading raises the question of whether they properly understand how to use and interpret statistics. Recent work suggests they do not. Nicholas Nassim Taleb has shown how most economic and financial models mistakenly assume that aggregation wipes out "fat tails," allowing orthodox economists to build models using well-behaved normal probability distributions. While financiers assume this transforms uncertainty into manageable risk, unsophisticated use of statistics causes this modeling process to fall into a tautological trap. When this circular logic is used to justify highly leveraged investment strategies, it yields a financial system shockingly vulnerable to rare but consequential "Black Swan events."

Macroeconomics' foundational problem: unemployment

Keynes (1936: 5) argues that the classical theory of employment relies on "two fundamental postulates" adopted "practically without discussion." This logical pair formalizes the decision-making calculus for the individual actors on each side of the wage bargain. The first postulate determines the demand schedule for hiring labor: "The wage is equal to the marginal product of labour." Through the logic of this first postulate, we see how entrepreneurs pass along deflationary pains to their workers in the form of wage cuts. The second postulate defines the supply schedule of labor by explaining how the individual worker decides for what wage he is willing to work. The postulate assumes the worker to be calculating a balance between "the utility of the wage when a given volume of labour is employed [and] the marginal disutility of that amount of employment. That is to say, the real wage of an employed person is that which is just sufficient (in the estimation of the employed persons themselves) to induce the volume of labour actually employed to be forthcoming." If this worker can find a job with a wage high enough to satisfy this calculation, he will pursue it. If he is not working, classical theorists assume that he must not think it to be worth

his while. Critically, the theory holds that any worker should be able to price himself into a job by settling for a lower wage. Thus, any apparent employment shortage is the fault of obstinate workers, who are unwilling to accept the equilibrium wage, not the result of impersonal economic forces.

Together, these postulates leave space for only two forms of unemployment, the "frictional unemployment" of workers "between jobs" and "voluntary unemployment." To the classical liberal, the latter situation is the result of an individual choice not to work for the prevailing wage and should be as acceptable to policy makers as any other rational decision. But for Keynes, writing at the height of the Depression's misery, this theory seemed crazy, "for, admittedly, more labour would, as a rule, be forthcoming at the existing money-wage if it were demanded." How could the classical theory fail to conceive of a third category – involuntary unemployment – when millions of the out-of-work were starving and searching without success for any available job? Classical theory explained away these masses' plights by blaming an "open or tacit agreement among workers not to work for less" (Keynes 1936: 5).

Inspired by the nonsensical conclusion that the Great Depression's astronomical unemployment was nothing more than millions of workers' simultaneous decisions that their jobs weren't worth the trouble, Keynes raises a fundamental objection to the classical theory of employment. The problem flows from an aggregation error. Extending the two postulates' conclusion (any worker can price himself into a job by lowering his wage demands) to the whole mass of unemployed workers leads not to jobs for all but rather to a now-familiar deflationary race to the bottom. The classical marketplace drama assumes, he argues, "that [workers] can, if they wish, bring their real wages into conformity with the marginal disutility of the amount of employment offered by the employers at that wage. If this is not true, then there is no longer any reason to expect a tendency towards equality between the real wage and the marginal disutility of labour" (Keynes 1936: 11). Of course, it is not true that the masses of workers have direct control over their real wages. Workers both negotiate and receive their income in the form of money wages, and "the struggle about money-wages primarily affects the *distribution* of the aggregate real wage between different labour-groups, and not its average amount per unit employment" (Keynes 1936: 14). As he fleshes out this objection to classical theory's second postulate, Keynes introduces a meaningful theoretical distance between individual decision making and the dynamics of the economy as a whole.

Crucially, a general reduction in real wages depends on a change in the purchasing power of money itself. Such a change in the cost of living is beyond the bargaining power of workers. That labour unions do not call for massive strikes with each increase in the cost of living (corresponding to a general decrease in real wages), testifies to workers' inability to affect any general change in their own purchasing power. In light of this disconnection, "a willingness on the part of labour to accept lower money-wages is not necessarily a remedy for unemployment" (Keynes 1936: 18). This move fundamentally changes the economic logic accounting for the existence of the industrial reserve army. Keynes' analysis frees workers and their collective bargaining organizations of the responsibility for chronic unemployment; their shortage of jobs is no longer an issue of individual obstinacy. In this regime, involuntary unemployment is symptom of a systemic imbalance in the economy as a whole: the aggregate labour supply exceeds the current aggregate demand for labour. Thus, Keynes establishes the management of unemployment as the foundational problem of macroeconomics.

Macroeconomics and physics

Political economy and physics have a history of intellectual exchange stretching back to the work of Aristotle, who might be said to have invented the philosophical study of both. The authors who built the foundations of classical liberalism – Locke, Smith, and Ricardo – had aspirations to do for their discipline what Newton had done for physics: discover a set of laws underlying all natural processes. The dynamical structure of liberal economic theory, in which prices converge on a central point of equilibrium, has clear inspirations in the Newtonian theory of gravitation. In terms of its method, logic, and inspirations, political economy might be called the "social physics" (Mirowski 1989).

There are strong justifications for subjecting Keynes' political economy to a comparative analysis with physics. His 1921 *Treatise On Probability* indicates his fluency with physics' latest problems, theories, and mathematics. Not only are many of his statistical methods shared with or derived by mathematical physicists, but the applications he discusses to illustrate the philosophical import of his work come from a variety of branches of physics, including thermodynamics, statistical mechanics, and astronomy (Keynes 1921: 630).

Indeed, the very title of his *magnum opus* alludes to the scientific revolution wrought by his acquaintance, Albert Einstein, in his recent generalization of relativity (Skidelsky 2003: 3).

Keynes' physical inspiration is clear from *The General Theory*'s first page. In this introductory statement, he states his intention to "argue that the postulates of the classical theory are applicable to a special case only and not to the general case, the situation which it assumes being a limiting point of the possible positions of equilibrium" (Keynes, 1936, p. 3) This correspondence principle, which implies that Keynes' general theory should reduce to the classical theory in the limit of full employment, could easily have been lifted from a physics textbook of the time. After Einstein ignited the twin projects of quantum mechanics and general relativity in his 1905 *annus mirabilis*, Bohr's correspondence principle – the idea that all new physical theories should reduce to familiar classical laws at the so-called Newtonian limits – served as a crucial guide and inviolable test for the surge of new concepts. Indeed, Keynes makes an oblique analogy to general relativity's new geometric theory of spacetime that reflects his aspirations for *The General Theory* to be on an intellectual par with modern physics:

> The classical [economic] theorists resemble Euclidean geometers in a non-Euclidean world who, discovering that in experience straight lines apparently parallel often meet, rebuke the lines for not keeping straight as the only remedy for the unfortunate collisions, which are occurring. Yet, in truth, there is no remedy except to throw over the axiom of parallels and to work out a non-Euclidean geometry. Something similar is required today in economics. We need to throw over the second postulate of the classical doctrine and to work out the behavior of a system in which involuntary unemployment in the strict sense is possible (Keynes 1936:.16-7).

If Keynes found *The General Theory*'s *raison d'être* in Einstein's general relativity, he found inspiration for its logical structure in the dual theories of thermodynamics and statistical mechanics. The slippery relation between these complementary theories of matter provides the perfect model by which one can grasp the radical connection between macroeconomic aggregates and Keynes' analysis of the individual psychology.

Thermodynamics rose from the residue of the caloric theory, which assumed heat to be a weightless fluid. This fluid, called caloric, could be neither created nor destroyed and flowed from hot bodies to colder ones. Temperature was thought to be a measure of the amount of caloric in a particular body. The theory was

essentially mechanical in nature, and its success benefitted greatly from the ease of imaging this fluid flowing back and forth like any familiar liquid. It was both causally simple and fully reversible.

Caloric theory evaporated in 1798, under the eye of Count Rumford (Benjamin Thompson), who was occupied with boring cannons for the Elector of Bavaria at his arsenal in Munich. As he watched his tools bore through the metal, Thompson observed the production of heat. Further experiments demonstrated that the friction of a dull tool against the metal produced an "evidently inexhaustible" supply of heat. This result contradicted the supposed conservation of caloric and set the stage for the invention of a substitute theory (Stehle 1994: 28). This came nearly half a century later, when a Manchester brewer, James Prescott Joule, conclusively showed heat to be a form of energy with a mechanical equivalent (Stehle 1994: 30).

In 1850, Rudolf Clausius formulated the modern laws of thermodynamics. The first law of thermodynamics is a mathematical statement of the conservation of energy, a principle that came directly from Joule's experiments, and is given by:

$$\Delta U = Q + W$$

In place of the once-conserved caloric, the first law stated that all bodies have some internal energy (U). Any change in this energy is the result of either adding some quantity of heat (Q) or performing some mechanical work on it (W). The law most readily applies to a gas-filled piston, which can gain energy by both heating and compression. To this, he added the second law of thermodynamics, a statement about entropy change, a newly defined ratio of added heat to temperature. From Sadi Carnot's study of steam engines, Clausius was able to state definitively that the entropy of any closed system cannot decrease. Further work on the entropy changes of steam engines by Lord Kelvin yielded the first rigorous definition of absolute temperature, thermodynamics' final important quantity (Stehle 1994: 30-2). These thermodynamic quantities – temperature, pressure, volume, and entropy – are *state variables*, which fully determine the behaviour of a quantity of bulk matter.

Although most of the crucial researches in the initial development of thermodynamics were performed on water, later work showed that the results are generally applicable. Moreover, the model of matter is also irrelevant: it wasn't until Einstein's 1905 explanation of Brownian motion that belief in the reality

of atoms and molecules was close to universal (Stehle 1994: 91). That thermodynamics is logically independent from any underlying theory of matter is crucial, because it speaks to thermodynamics' intrinsically totalizing register of analysis. It is a science of aggregates and equilibria, working with state variables. Without relying on any further levels of description, thermodynamics can predict the evolution of whole physical systems through intermediate equilibria. Its equations can calculate the progress of a chemical reaction, even without any knowledge of atoms, molecules, or elements.

Keynesian macroeconomics operates on an analogous register. As we have seen, it takes its object to be the economy as a whole. The individuals who labour, invest, and consume within the economy are not, in themselves, causally responsible for macroeconomic performance. While classical political economy sees unemployment as the result of many workers' voluntary decisions not to work, perhaps summed in organized labor's unwillingness to agree to supposedly necessary wage cuts, Keynesian macroeconomics accounts for the reality of involuntary unemployment with the gap between effective demand and the level of output necessary to support an economy's current productive capacity. As a source of policy, macroeconomics aims to stimulate growth with additional investment demand or temper an overheated economy with consumption taxes. Macroeconomics does not take individuals, in the liberal sense of the term, as objects of its attention or theoretical terms in its calculations. Furthermore, the quantities that serve as state variables in macroeconomic equations (e.g. unemployment rate) only have meaning in relation to the whole economy and lack microeconomic interpretations.

So how does individual behavior link up with macroeconomic aggregates? Macroeconomics understands effective demand to be the primary driver of economic activity. Successful management of unemployment depends on not only the maintenance of healthy consumer demand but also continual increases in investment demand to fill the ever-widening savings gap between consumption and the level of production necessary to support full employment. For each of these components of aggregate demand, *The General Theory* provides a rate-determining function: the marginal propensity to consume (MPC) and the inducement to invest, respectively. Classically, Say's Law guarantees that supply will create its own demand because it assumes "that the aggregate demand price of output as a whole is equal to its aggregate supply price for all volumes of output" (Keynes 1936: 26). Insofar as it implicitly fixes the MPC's value to unity,

Classical theory is "a special case only and not... the general case, the situation which it assumes being a limiting point of the possible positions of equilibrium" (Keynes 1936: 3). In contrast, Keynes considers them the results of complex integrations of psychological tendencies, objective circumstances, and predictions about the future.

It is because the MPC almost always has a value below unity that consumer demand alone is insufficient to support full employment. The MPC's "normal shape" follows from a "fundamental psychological law... that men are disposed, *as a rule and on the average*, to increase their consumption as their income increases, but not by as much as the increase in their income.... For a man's habitual standard of life usually has the first claim on his income, and he is apt to save the difference which discovers itself between his actual income and the expense of his habitual standard" (Keynes 1936: 96-7). While all economic theories make claims about human nature, the distinctively Keynesian logic is encapsulated in the phrase "as a rule and on the average." Keynesian appeals to psychological laws never yield rigid decision-making calculi; any conclusions are mere tendencies. Instead of mathematically determining what actions a rigorously rational individual should take, these conclusions merely suggest, "as a rule and on the average" what people will do. This psychological theory is fundamentally statistical in nature and the resulting macroeconomic quantities do not supervene on individual patterns of thought in any deterministic way.

To understand how *The General Theory*'s macroeconomics is truly irreducible to individual psychology, we must return to the nineteenth century and investigate the statistical physics that began to emerge from thermodynamics. It began with a controversial theory of atoms. By observing chemical reactions, Amedeo Avogadro had concluded that any given volume of gas at a standard temperature and pressure consisted of the same number of molecules, regardless of the type of substance. During 1850s, this proposal began to attract rigorous analysis and, in the process, became known as the kinetic theory of gases. In 1851, Joule worked out an explanation for gaseous pressure, building on some century-old work by Daniel Bernoulli. Joule calculated the speed at which some hypothetical and indefinitely small gas particles must move so that their collisions with the walls of a container could produce a definite pressure. A container of hydrogen gas at atmospheric pressure corresponds, he argues, to a vast number of particles striking the walls at 6225 feet per second. He calculates this with an arithmetic combination of state and particulate variables, while assuming that the gas is

composed of only three particles (Joule 1857). This analysis directly connects pressure – a thermodynamic quantity describing the state of a gas as a whole – to the motions of its constituent particles.

Although the kinetic theory of gases grounded thermodynamics in an emerging atomic theory of matter, it is still a wholly classical theory. In deducing the speed of molecular motions from gaseous pressure, Bernoulli, Joule, and Clausius each implicitly assumed that particles composing a gas behaved uniformly. From any given temperature and pressure information, Joule calculated a single speed, which he thought described the motions of all the molecules in the gas. Kinetic theory fully determines the relationship between this uniform molecular motion and a gas' thermodynamic behaviour. Since many physicists of the day were sceptical that matter is composed of atoms and the only available measurements were of thermodynamical quantities, kinetic theorists considered molecular speed to supervene on the temperature and pressure of the gas as a whole.

In 1859, James Clerk Maxwell disrupted this straightforward determinism by inaugurating a robust statistical approach to the problem of molecular motion. His groundbreaking "Illustrations of the Dynamical Theory of Gases," read in front of the British Association for the Advancement of Science in Aberdeen, Scotland, contained several significant advances, including derivations of the equipartition of energy, the viscosity of gases, and the conduction of heat by gases (Stehle 1994: 36-7). But the paper's most important result was a statistical distribution describing non-uniform molecular velocities in a gas at a given temperature and pressure. Proceeding probabilistically, Maxwell severed the rigid mathematical link between thermodynamics and the motion of an individual molecule.

The leaders in kinetic theory – Bernoulli, Joule, and Clausius – had already modelled gaseous pressure as the cumulative force of molecules colliding with the walls of a container. Clausius had also calculated the mean free path gas molecules traverse before colliding with each other. This tiny distance explains why gases are rarely observed moving through the world at thousands of feet per second. Maxwell assumed that after many collisions, gas molecules' directions of motion would be isotropic, a relatively weak statistical claim. So, in modelling a collision between two molecules, Maxwell had to account for a complete uncertainty as to the details of the scattered particles' incoming trajectories. His paper begins with a geometric argument that describes the possible scattering

cross sections for two colliding gas molecules in completely general terms. From here, he finds the probability of the particles rebounding in a given range of directions, and resolves this velocity into rectilinear components. In this step, he assumes that the probability of a molecule's velocity along one coordinate axis is independent of its motion along the other two. Later, he would be forced to abandon this assumption and couple the equations for each component of the particle's velocity. (We will soon see the importance of this rejected assumption.) Continuing with his probabilistic analysis, Maxwell deduces that the number of particles whose actual velocity lies between the limits *v* and *v* + *dv* is

$$ N \frac{4}{\alpha^3 \sqrt{\pi}} v^2 e^{-\left(v^2/\alpha^2\right)} dv \; . $$

According to this function, all molecular velocities from 0 to ∞ are possible, but they are distributed among the many particles according to this well-defined law. He goes on to derive probabilistic relationships between the parameters of this velocity distribution and the familiar thermodynamic state variables (Maxwell 1860). Notice that Maxwell attends to uncertainty from the first step of his reasoning. His probabilistic logic always accounts for those mechanical details that are necessarily unknowable.

As Maxwell continued to refine his theory, others began working on the statistical problem. Chief among them was Ludwig Boltzmann, whose "Further Studies on the Thermal Equilibrium of Gas Molecules" generalizes Maxwell's results and solidifies their foundations in probability theory. Boltzmann begins his paper by directly addressing the apparent incongruity between thermodynamics' fully determined equations describing warm bodies and the complete molecular randomness suggested by Maxwell's statistical mechanics:

> The mechanical theory of heat assumes that the molecules of a gas are not at rest, but rather are in the liveliest motion. Hence, even though the body does not change its state, its individual molecules are always changing their states of motion, and the various molecules take up many different positions with respect to each other. The fact that we nevertheless observe completely definite laws of behavior of warm bodies is to be attributed to the circumstance that the most random events, when they occur in the same proportions, give the same average value. For the molecules of the body are indeed so numerous, and their motion is so rapid, that we can perceive nothing more than average values. One might compare the regularity of these average values with the amazing constancy of the average numbers provided by statistics, which are also derived from processes each of which is determined by a completely

unpredicatable interaction with many other factors. The molecules are likewise just so many individuals having the most varied states of motion, and it is only because the number of them that have, on the average, a particular state of motion is constant, that the properties of the gas remain unchanged. The determination of average values is the task of probability theory. Hence, the problems of he mechanical theory of heat are also problems of probability theory (Boltzmann 1872).

Boltzmann grasped probability theory's power to resolve micro-scale indeterminacy into definite macro-scale dynamics better than anyone. He goes on to re-derive Maxwell's result in even greater generality, extending the analysis from spherically symmetric mono-atomic molecules to polyatomic molecules of any shape. Although the problem of polyatomic molecules introduces integrals that cannot be solved analytically, Boltzmann finds motivation in the thermodynamical laws' logical independence from such microscopic concerns. In general, self-interacting physical systems tend to be non-linear, preventing simple scaling from micro- to macro-scale descriptions. Indeed, Boltzmann's generalization goes so far as to prove that no matter what starting conditions are assumed – even uniform molecular motion – an ensemble of mutually repulsive gas particles will always tend toward the Maxwellian velocity distribution. Thus, the statistical form of this distribution is an intrinsic feature of a self-interacting, many-body system and not particular to any set of specific assumptions about gas molecules.

Maxwell's velocity distribution marks statistical mechanic's emergence from the wholly classical kinetic theory of gases, establishing two distinct registers on which physicists model matter. Statistical mechanics shows how the dynamic state of a system measured at the macro-level does not seamlessly scale down to reveal action at the particle level. Though it might have reached a steady thermodynamic equilibrium, this does not imply the ability to precisely describe the motion of any individual gas particle. In contrast, the kinetic theory of Bernoulli, Joule, and Clausius imagines a container of gas to be filled with spherical particles bouncing off the walls and each other and all travelling at the same speed, albeit in random directions. According to kinetic theory's non-statistical mathematics, the gas' temperature, pressure, and volume data uniquely determine that speed. There is no room for any meaningful uncertainty. Similarly, classical political economy permits a perfectly continuous scaling between the microeconomics describing individual market actors and the macroeconomic behaviour of state-sized economies. Insofar as Say's Law

guarantees that some demand will emerge in response to any supply brought to market, potential investors face no uncertainty in funding a well-managed firm. In such a certain world, an economy would have an infinite capacity for job creation, and so individuals should have no trouble finding work, as long as they are willing to accept the equilibrium's market-clearing wage.

Like Maxwell, Keynes replaced a set of fully determinate laws with a fundamentally statistical theory, in which the motion of aggregate quantities are irreducible to large but simple sums of individual decisions. This demonstrates the need for a macroeconomics without microfoundations, one that takes the economy as a whole to be a real, organic object of study. Both thermodynamics and Keynes' macroeconomics confirm this epistemological rupture with their independence from particular theories of microscale mechanics. Just as one can use thermodynamics to calculate the equilibrium concentrations in a chemical reaction without reference to atoms or molecules, Keynes' *General Theory* shows how classical political economy faces aggregation problems even before sacrificing the comprehensive rationality of the liberal subject. A liquidity crisis is only the most dramatic example of how many individually rational decisions combine to produce an irrational market failure. Indeed, Keynes argues that on a macroeconomic level, liquidity is an illusion. "Of the maxims of orthodox finance none, surely, is more anti-social that the fetish of liquidity, the doctrine that it is a positive virtue on the part of investment institutions to concentrate their resources upon the holding of 'liquid' securities. It forgets that there is no such thing as liquidity of investment for the community as a whole." (Keynes 1936: 155) Such "rational irrationality" demonstrates why the economic whole must be understood to be more than just a sum of its individual parts (Cassidy 2009). Aggregating individual decisions produces effects on its own, effects that are invisible if one naively assumes that rationality scales continuously.

Grasping the individual in macroeconomics

Keynes does not settle for the classically assumption that market actors are perfectly rational individuals. Instead, he pairs macroeconomics with a theory of human psychology in which uncertainty, rather than omniscient rationality, is the overwhelming force in individual decision-making. Rationality is a theoretically attractive assumption because it mandates that individuals in a

given situation act uniformly. This keeps neoclassical economic questions mathematically tractable, just as Joule's implicit assumption of a uniform molecular speed kept his algebra simple. But when uncertainty about the future dominates decision-making, people can no longer be modelled as moving in lockstep, and their decisions are no longer rigidly determined by known macroeconomic conditions. It is the challenge of handling people's varying expectations of the future that requires macroeconomics to include individuals only by means of statistical laws.

As discussed above, individual psychology enters into macroeconomic equations through two rate-determining functions: the marginal propensity to consume and the inducement to invest. Neither measure is derived from the hypothetical decisions of a single individual, as it would be in a similar classical case. Recall that "as a rule and on the average," Keynes defines consumption as a function of income. Thus, any single MPC value entered into macroeconomic calculations corresponds to a wide distribution of consumption rates. This statistical effect is a non-trivial addition to economic theory; the MPC reflects the distribution of incomes in a country, which determines the size of the effective demand gap investment must fill to support full employment.

If Keynes' macroeconomics operates on an aggregate register similar to thermodynamics', then his theory of human psychology is analogous to statistical mechanics. This becomes clear in his analysis of the inducement to invest, which takes place "on a different level of abstraction from most of [his] book." Keynes (1936: 147-9) argues that firms make investment decisions on the basis of their long-term expectations for prospective yields. The state of these expectations "does not solely depend on the most probable forecast we can make. It also depends on the *confidence* with which we make this forecast – on how highly we rate the likelihood of our best forecast turning out quite wrong." With confidence as such an essential feature, mathematically modelling the inducement to invest for macroeconomics requires a theory of probability structured around the way that people actually make decisions in the face of fundamental uncertainty.

The orthodox treatment of economic decisions holds that all rational actors possess a utility function and defines rational behaviour as making decisions that will maximize its expected value (Runde 2000: 216). This expected value hypothesis, which encodes a natural risk aversion into the utility function, can be traced to Bernoulli's 18th century work on probability (Bernoulli 1738). In

their canonical work, *Theory of Games and Economic Behaviour*, John von Neumann and Oskar Morgenstern aim "to find the mathematically complete principles which define 'rational behaviour' for the participants in a social economy.... The immediate concept of a solution is plausibly a set of rules for each participant which tells him how to behave in every situation which may conceivably arise." Although the difficulties of generalizing this approach force them to broaden the notion to a set of sets of rules they call "standards of behaviour," their philosophical goal remains "an absolute state of equilibrium in which the quantitative share of participants would be precisely determined." To move from simply defined games to more complex decision-making environments, von Neumann and Morgenstern (1944: 31-5) statistically define the background of economic activities. This move permits them to determine probabilities for the consequences of various decisions and encode this information in an agent's VNM utility function. Although describing a population with rigidly defined rational choices seems to strongly suggest the emergence of aggregate level coherence effects, they are invisible to microeconomics; only macroeconomics' analysis of the economic whole can reveal the rational irrationality of bank runs, fire sales, and liquidity crises. Even when individuals are modelled as behaving differently, according to a range of possible preferences and appetites for financial risk, orthodox theories still model a given decision of whether or not to invest as a forced choice, determined by the actor's personal utility function. Although people's behaviours might not be strictly uniform, orthodox economic and financial models still produce only a "mild randomness" derived from the physical theory of Brownian motion, which econometricians describe with normal distributions. However, Mandelbrot has empirically demonstrated not only that the behaviour of large-scale markets is not Gaussian and the movements of asset prices are not Brownian but also that these simplifying assumptions produce dangerously inaccurate models of markets (Mandelbrot 1963).

G.L.S. Shackle criticizes the orthodox approach for misapplying standard probability calculus to human decision-making. The crux of his critique focuses on Von Neumann and Morgenstern's reliance on the "perfectly well founded interpretation of probability as frequency in long runs" (von Neumann and Morgenstern 1944: 19). This "frequency ratio" interpretation defines the probability of a given consequence with a ratio that equals the proportion of its outcomes after a large number of experimental trials. Logically, the probability ratios of all possible outcomes must sum to unity, as in the simple case of a dice

game. But, Shackle argues, investment opportunities are "indivisible experiments," which can never be repeated because the decision of whether or not to invest destroys those specific conditions. In the context of these one-shot decisions, "it does not make sense to assign even subjectively-determined 'frequency-ratios' to the various hypotheses, multiply, and add the products. The result will be logically meaningless." Furthermore, an agent facing true uncertainty is often unable to make an exhaustive list of all possible outcomes of a decision, so it is unreasonable to demand that all probabilities sum to unity (Shackle 1949: 70-4). The supposedly well constructed risk models for financial markets notwithstanding, empirical research has shown that price changes of nearly all assets scale dramatically, so potential losses from an investment must be considered practically unbounded (Mandelbrot 2004: 62-72). Thus, it is practically impossible for an actor's expectations to be comprehensive.

Instead, Shackle proposes an alternative, "potential surprise" interpretation of probability that better reflects the outsized role of confidence in the psychology of human expectations. In this scheme, probabilities are derived from the potential surprise an individual would experience should an event come to pass. An agent determines this "purely subjective" measure of potential surprise by comparing how well possible outcomes match his information about the state of the world. Note that the Keynesian notion of confidence is endogenous to a subject's judgment of potential surprise. Because these probabilities are in a sense intrinsic to each hypothesis, they do not have to add to unity. This permits an agent to judge that several different hypotheses – even mutually exclusive hypotheses – all have zero potential surprise, so long as they follow from what he knows about the present state of the world (Shackle 1949).

Thus, Shackle argues that "our power to form expectations is limited to *excluding* some of the conceivable outcomes by associating with them varying degrees of potential surprise, leaving at the core either a unique outcome or *a range within which there is complete, unqualified indeterminacy*" (Shackle 1940). In this case, agents must decide whether or not to invest on some basis of attraction other than likelihood. Because clear calculation requires actual values, Shackle writes, "it is the *extreme* values, from amongst those regarded as all equally possible or not potentially very surprising, on which entrepreneurs' attention will be focused, and that he will *assume* the best as long as the worst is not too bad" (1940). This reduces an agent's decision to a comparison of whether his anticipation of high returns outweighs his fear of the worst anticipated outcome.

When economists interpret probability according to Shackle's potential surprise scheme rather than the expected value hypothesis, the inducement to invest becomes a poorly behaved function. As agents acquire new information, the content and clarity of their expectations shift, subjecting their inducement to invest to "large *discontinuous* changes" (Shackle, 1940). These expectations are not coldly calculated forecasts but rather depend on agents' confidence in the accuracy of their predictions as well as their optimism about the state of the economy in general. Indeed, expectations are "more the momentary creation of the latest news than a stable and gradually modified structure: they are generated afresh from moment to moment rather than continuously evolved" (Shackle 1940). The combination of these "sudden and radical changes in the content of expectations" and the inherent variation in people's "animal spirits" saps stability from the inducement to invest (Shackle 1940; Keynes 1936: 161). Since firms always have the option of postponing investment to wait for more information, small decreases in their confidence can encourage them to put off spending. But insofar as uncertainty never vanishes completely, agents will always have the option of not spending at the present. Shackle argues that this perpetual option will depress the inducement to invest below the level suggested by the macroeconomic content of expectations alone (Shackle 1940).

A characteristic that is so discontinuous on the individual level can only be represented on the aggregate level using a robust statistics. Just as statistical mechanics connects the randomly moving particles in a gas to a thermodynamic equilibrium, Keynesian macroeconomics must transform the effectively random distribution of individuals' inducements to invest into a rate determining, aggregate quantity. The macroeconomic function is irreducible to the result of a thought experiment about rationality, since there is no single representative subject to consider. Conversely, given an aggregate rate of investment and other macroeconomic data, it is impossible to precisely determine any individual's inducement to invest. According to von Neumann and Morgenstern's theory of utility maximization, however, any agent facing a given background of economic data has a mathematically defined rational choice. This rigid determinism is absent from Keynesian theory.

Furthermore, the ubiquitous discontinuities in an inducement to invest modelled with Shackle's potential surprise interpretation of probability finds empirical support in Mandelbrot's analysis of market prices. One of the central tenets in his critique of orthodox econometrics attacks the fundamental assumption that

asset prices vary continuously; they don't, he argues. In fact, he reports that the most statistically important changes in most assets' price series are dislocations or jumps, and that these discontinuities invalidate the common statistical tools of orthodox econometrics and finance, such as the Black-Scholes pricing model (Mandelbrot 2004: 237). Insofar as asset prices are derived from the interaction of an investment market's supply and demand, which are, in turn, functions of the reigning inducement to invest, Shackle's theory of decision-making by potential surprise might account for the discontinuities Mandelbrot observes.

A Keynesian epistemology of statistics

Keynes was not only a celebrated economist; he was also a respected mathematician. After earning his first class B.A. in mathematics from Cambridge, his mathematical work focused on statistics, culminating in his 1921 *Treatise on Probability*. It is a substantial text both mathematically and philosophically, in which he works out several new probabilistic methods and develops a subtle epistemology of statistical laws. Reading Keynes' *Treatise* alongside *The General Theory* reveals the extent to which his macroeconomics relies on some fundamental philosophical insights about the interpretation of probabilities. Furthermore, the *Treatise* supports this essay's reading of *The General Theory's* dual structure, as Keynes' philosophy of probability emerges from his analysis of physical theory, particularly his work on Maxwell's velocity distribution. This suggests that the theoretical similarity between Keynesian macroeconomics and the complex of thermodynamics and statistical mechanics is no mere coincidence but a definite genealogy.

In his *Treatise*, Keynes performs a lengthy analysis of "Maxwell's classic mistake in the theory of gases," thoroughly critiquing the physicist's mistaken assumption that the components of a gas molecule's velocity are independent of each other. Keynes' analysis is no mere recapitulation of others' work; he rejects the conclusions of three "authorities" (Bertrand, Poincaré, and Von Kries) and forges his own explanation for Maxwell's statistical miscue. Keynes' critique focuses on the fundamental uncertainty in the calculation of a particle's velocity. He stresses that the mathematical deduction of its mechanics necessarily works only from components to total velocity and not vice versa. In other words, Maxwell assumed too much determinism in the knowledge of

higher-order data; aggregated data do not break down to their components as directly as he initially thought. In sum, this section of the *Treatise* reveals that Keynes was as fluent as anyone in both the physics and the mathematics of statistical mechanics. Furthermore, his critique was motivated by what he saw as the intrinsic impossibility of transferring certainty freely through a statistical relationship (Keynes 1921: 172-4). This view is very much in line with Shackle's approach to modelling expectations.

Keynes considers probabilities to be indications of the certainty associated with any piece of knowledge, mediating the relation between those things with which we have "*direct acquaintance*" and propositions connected to them "indirectly, *by argument.*" Probabilities allow subjects to pass by argument from one proposition to another "without being able to say what logical relations, if any, we have perceived between them." Thus, Keynes (1921: 10-3) argues that probability is a logical tool with which reason can extend its analytical power in the face of fundamental uncertainty. This robust epistemology should be distinguished from the weak, frequency-ratio interpretation, which takes probability to be a tool used in placing bets on repeating events with a known distribution of outcomes (e.g. card or dice games). Instead, probabilities permit subjects to know, with some degree of confidence below certainty, in situations when arguments hold together chains of propositions that are not fully determined. By allowing these jumps from one level of abstraction to another, probability calculus enables theoreticians to articulate dynamical relations in cases when irreducible uncertainty prevents them from knowing the details of an underlying causal chain. Probabilistic analysis is an inferential tool that forms connections between analytical levels without collapsing them into each other. It gives theoreticians the confidence necessary to infer such relationships without articulating their details with rigorous logic.

Formulating high-level dynamics in the absence of a complete causal story suggests the reality of emergent properties and their logical independence from lower-level mechanics. To Keynes, this means rejecting

> something much more like what mathematicians call the principle of the superposition of small effects or as I prefer to call it... the *atomic* character of natural law.... Each atom can, according to this theory, be treated as a separate cause and does not enter into different organic combinations, in each of which it is regulated by different laws. Perhaps it has not always been realized that this atomic uniformity is in no way implied by the principle of the Uniformity of Nature. Yet

> there might well be quite different laws for wholes of different degrees of complexity, and laws of connection between complexes, which could not be stated in terms of laws connecting individual parts. In this case natural law would be *organic* and not, as it is generally supposed, atomic (Keynes 1921: 249).

Of these two possible meronomies, it is clear that truly statistical sciences take the world to consist of organic wholes. In a fundamentally atomic world, addition would be sufficient to derive how a large number of atoms would *cause* in concert. The power of statistics is its ability to transform an ensemble as new object with an existence all its own, bridging the explanatory gap between micro- and macro-scale behavior. Indeed, both macroeconomics and thermodynamics/statistical mechanics are premised on such a disconnection between the aggregate and the particulate. In both domains, the dynamics of "organic" aggregates are irreducible to the sum of individual interactions between people or particles. There is an implicit ontological claim here: Keynesian statistical analysis *creates* the macroeconomy as a new object, insofar as macroeconomics directs policy proposals at it. The existence of "unemployment policy" and government stimulus spending confirms that the macroeconomy does, in fact, exist.

Conclusion

This reading suggests that Keynes' *General Theory* did not so much add ideas onto the existing edifice of classical political economy as initiate a profound epistemological break from it. Keynesian macroeconomics is the first theory that takes the economy to exist as an organic whole in its own right. In this view, aggregation requires robust statistical methods, not simple means and variances as classical theory assumes. According to Keynes' critique, classical theory's faith in the power of markets to optimally allocate resources on the basis of decentralized self-interest relies on a fallacy of composition:

> The possibility of our knowing that one thing rather than another is our duty depends upon the assumption that a greater goodness in any part makes, in the absence of evidence to the contrary, a greater goodness in the whole more probable than would the lesser goodness of the part. We assume that the goodness of a part is favourably relevant to the goodness of the whole. Without this assumption we have no reason, not even a probable one, for preferring one action to any other on the whole. If we suppose that goodness is always organic, whether the whole is composed

> of simultaneous or successive parts, such an assumption is not easily justified. The case is parallel to the question, whether physical law is organic or atomic (Keynes 1921: 310).

Keynes disproves the premise that goodness adds arithmetically by pointing out the harmful economic effects of aggregation problems. Classical theory can explain neither the rampant and involuntary unemployment of a recession nor the liquidity crisis that results from a general loss of confidence, but *The General Theory* shows how this macro-scale coherence develops from the aggregation of individually rational strategies. In both cases, goodness for the part leads to an irrational result for the whole.

While millions of investment and consumption decisions can drive an economy through wrenching booms and busts, these aggregate-level dynamics are irreducible to the analysis of individual decision-making. Instead of resorting to classical Gedanken experiments to determine a hypothetical individual's most rational choice in a given economic situation, Keynes' theory analyzes the psychology of expectations in an environment of fundamental uncertainty. Mathematically modeling Keynesian expectations required a new probability calculus that eschews the conventional frequency-ratio interpretation of probability undergirding the expected utility hypothesis. GLS Shackle's approach handles one-off decisions by deriving probabilities from a subject's potential surprise should an event come to pass. His analysis of investment decisions shows how an individual's inducement to invest is liable to experience large, unpredictable discontinuities in the face of an uncertain future. Thus, macroeconomics cannot glean the aggregate rates of investment or consumption by analyzing a single agent's rational choice. Instead, it must assume that there is a wildly random distribution of individuals' expectations and derive a stable aggregate value by robust statistical methods, without assuming *a priori* that market movements are continuous and normally distributed. This difference between classical and Keynesian economics is akin to the distinction between the fully determinate kinetic theory of gases and statistical mechanics. Indeed, Keynes' *Treatise on Probability* suggests that he found inspiration for his epistemology in Maxwell's theoretical intervention on behalf of uncertainty and randomness.

Insofar as Keynes' epistemology lies at macroeconomics' logical core, the neoclassical project to supply it with microfoundations is incoherent on its face.

A firm belief in what Keynes called "the atomic character" of economic theory is an essential component of the rational expectations school's critique of Keynesianism:

> Aggregate behaviour in Keynesian models does not correspond with individual optimizing behaviour in all conditions. It is, at best, consistent with individual behaviour only under some specific conditions.... The rational expectations school maintains that only by formulating in a coherent way the decision problem facing individuals can one begin to develop models capable of evaluating policy correctly. *Because aggregate outcomes are only a sum of individual decisions, the aggregate relationships should have no independent existence*, but they do under the Keynesian approach (Willes 1981:. 367-8).

On the basis of the reading presented above, this rational expectations critique fundamentally misunderstands the logic of macroeconomics. It is precisely this belief in aggregate relationships' organic, independent existence that permits macroeconomics to see involuntary unemployment and grasp the rational irrationality of liquidity crises. One need not even address the question of whether – in light of *The General Theory*'s comments about uncertainty – expectations about the future can ever be rational in order to see that the concept of a macroeconomics with microfoundations is inherently contradictory. Microeconomic analysis alone cannot account for the economy-wide coherence of individuals' behaviour, which is responsible for a bank run. This limitation is analogous to the difficulty of representing phase transitions in matter. Physicists can only describe the transformation of a liquid into a gas with thermodynamics; there is no purely microscopic account.

Finally, we return to the question of the place of statistical methods in contemporary econometrics and finance. Considering the extent to which economists are enamoured with regression analysis, the calculation of moving averages, and other statistical methods, how can this essay claim that mainstream economics is insufficiently statistical? The answer is that it is not the mere invocation of statistics that matters, but the unsophisticated use to which these methods are put. Unsupported by a well-developed epistemology, statistics provide only the false reassurance of mathematics while leaving expert analyses vulnerable to dangerous tautology.

In his *Treatise*, Keynes cautions against the common error of slipping heedlessly between statistics' two functions: description and induction. Though an economist begins his analysis by fitting a certain statistical distribution to a

given set of random data, problems arise when he seeks to extend his description into a forecast without a rigorous transitional argument (Keynes 1921: 367-8). Recent work has shown how this kind of sloppy inference is endemic to modern econometric and financial models. In light of the 2008 financial crisis, there has been growing interest in the problem of "fat tails" in statistical distributions. Taleb, working with a detailed analysis of economic data representing nearly 98% of the globe's tradable volume, shows how most financial and econometric forecasts crucially rely on a set of classical assumptions, which are patently unjustifiable. These assumptions ignore the rare but important contributions of distributions' "fat tails" to overall economic performance, leading necessarily into tautologies. While modelers believe they have transformed uncertainty into manageable risk, the result is a fundamentally flawed inductive method that leaves models vulnerable to these "Black Swan events" (Taleb 2009).

Unwilling to execute trading strategies in a truly uncertain environment, financiers attempt to transform Knightian uncertainty into risk, which is "a quantity susceptible of measurement." Since models facing risk can incorporate estimates of their own error into forecasts, risk "is not in effect an uncertainty at all" (Knight 1957: 20). Taleb, however, argues that these models merely provide an illusion of safety, since risk managers cannot calculate – and thus, cannot consider – the probability of a rare but catastrophic "Black Swan" event. Risk managers craft forecasts by fitting a known probability distribution to a data set produced by an unknown generator. This requires them to make an assumption as to the general form of the distribution (e.g. Gaussian, Poisson, binomial) and then adjust its parameters until it closely matches the historical data. But the accuracy of this process cannot be validated, for it falls quickly into tautology:

> In almost all important cases, whether in the "hard" or "soft" sciences, the generator [of events] is hidden. There is no independent way to find out the parameters – e.g. the mean, standard deviation, etc. -– of the generator except for trying to infer it from the past behavior of the generator. On the other hand, in order to give any estimate of these parameters in the first place, one must first assume that the generator in question is of a certain general type: that it is a Normal generator, or a Poisson generator, etc.... We claim that most situations risk managers deal with are just such "bad" cases where one cannot figure out the general type of generator solely from the data, or at least give worthwhile estimate of its parameters. This means that any relation between the risks they calculate for "black swan" events, and the

actual risks of such events, may be purely coincidental. We are in uncertainty: we cannot tell not only whether or not X will happen, but not even give any reliable estimate of what $P(X)$ is (Taleb and Pelpel 2004: 6-7).

Although most models rely on such an *a priori* assumption that an invisible generator is best described by a probability distribution with a well-behaved, general form, Taleb's analysis shows that that this is little more than wishful thinking. In fact, "fat tails" dominate the historical returns of most economic indices, and the apparent universality of these common distributions is more a product of our modelling choices than the natural order of things. Conventional economics assumes that all data will converge according to the central limit theorem under summation. This simplistic view of aggregation is rooted in the familiar misunderstanding of statistics; even when it acknowledges a distribution of data, it assumes a long-run convergence justifies focusing solely on the mean and ignoring the tails. However, Taleb found "no evidence of 'convergence to normality' by aggregation.... The fatness of the tail seems to be conserved under aggregation." (Taleb 2009)

Contrast this with Maxwell and Boltzmann's strategy; they derive the form of the distribution rather than asserting the fit *a priori*. Here, the statistics are built up from a probabilistic analysis of individual collisions' geometry to infer a relationship with the thermodynamic whole. As with Shackle's potential surprise theory of probability, the particulate-level analysis can only glean limits – either the radii of gas molecules or the range of equally unsurprising expectations – within which uncertainty reigns.

The result of this epistemological shortcoming is that mainstream economics conceives a world that is far smoother and more manageable than reality. Taleb argues that the assumption of long-run normality is both naïve and dangerous. In a world of hidden generators and complex, unbounded payoffs (his so-called "Fourth Quadrant") conventional statistics are impotent (Taleb 2009). While financiers believe they have converted uncertainty into risk by calculating means and standard deviations, Black Swan events make this impossible. Instead, he suggests using investment strategies that simplify payoff structures as a way to ward off the disastrous effects of unpredictable Black Swan events.

Similarly, Mandelbrot's trenchant critique of the mathematics undergirding neoclassical econometrics and orthodox finance suggests that markets are far more statistically complex, far more random, and far riskier than commonly

assumed. The probability theory behind the efficient market hypothesis and its derivative analytical tools relies on fundamental assumptions about the Gaussian normality of aggregated behaviour, the continuity of prices, and the statistical independence of their changes to make markets mathematically tractable. Yet Mandelbrot has demonstrated that none of these assumptions holds up to empirical evidence. The result is an econometrics that invalidly smoothes over the wild randomness of markets, so that the economy appears more stable and financial markets less risky than they actually are. Mandelbrot advocates replacing these failed ideas with a statistics founded on his multifractal analysis, which requires none of the current theories' invalid assumptions and produces models that more accurately resemble really existing markets (Mandelbrot 2004: 225-76).

Multifractal analysis' alternative assumptions mesh well with those of the macroeconomics described above and seem to be a promising statistical alternative to the failures of neoclassical theory. We have already seen how the discontinuities intrinsic to Shackle's potential surprise theory of investment might account for the unexplained discontinuities present in Mandelbrot's fractal generators. In addition, neither Keynesian macroeconomics nor Mandelbrotian multifractal analysis assumes the economy to be populated with ideal rational agents whose investment choices are fully determined by their preferences and macroeconomic contexts. They also share a view that aggregating individuals' decisions into a large market produces its own effects. Rather than assuming that aggregation smoothes away outliers and dismissing crashes as statistical freaks or financial acts of god, both multifractal analysis and Kenyesian macroeconomics take large economic systems to have an intrinsic statistical complexity. Finally, on a more philosophical level, both Keynes' and Mandelbrot's critiques share a similar motivation: describing "the economic society in which we actually live" (Keynes 1936: 3).

In an irreducibly complex environment, there is no reliable way to transcend uncertainty and live in a world of entirely manageable risk. But even if individual firms' risk management models suggested the right mixes of credit default swaps, options, and insurance, the financial crisis of 2008 should serve as a warning that investment risk also resists straightforward aggregation. Despite – or rather because of – these redundancies, Black Swan losses reverberated through the financial system, threatening firms far removed from the collateralized debt obligations at the heart of the crisis. Just as the Depression's

bank runs proved that "there is no such thing as liquidity of investment for the community as a whole," the recent financial crisis should teach policymakers that there is no such thing as risk management for the whole economy (Keynes 1936: 155).

These fallacies of composition are endemic to a weak epistemology premised on a belief in "the atomic character of natural law" (Keynes 1921: 249). Maxwell and Boltzmann showed how a self-interacting, many-body system tends to behave as an organic whole and introduced uncertainty into the physics of matter. Keynes takes this statistical epistemology further, building a theory of the economy itself, irreducible to optimized individual decision-making. Aggregation is not a smooth diffusion of values around a central limit, according to a normal distribution. Instead, an organic macroeconomy exists in its own right, and it is prone to unexpected crises. In the face of hidden probability generators' fundamental uncertainty, aggregation leads to rare but potent concentrations, so that the individually rational sums to the collectively insane.

Acknowledgements

References

Bernoulli, D. (1738), 'Exposition of a new theory on the measurement of risk", trans. Louise Sommer, *Econometrica*, 22 (1), Jan 1954, 22-36.

Boltzmann, Ludwig (1872), "Further studies on the thermal equilibrium of gas molecules", originally published under the title "Weitere Studien über das Wärmegleichgewicht unter Gasmolekülen," in *Sitzungsberichte Akad. Wiss.*, Vienna, part II, 66, pp. 275-370 (1872); reprinted in *The Kinetic Theory of Gases*, ed. Stephen G Brush, pp. 262-349.

Cassidy, John (2009), 'Rational irrationality'. *New Yorker*, October 5.

Joule, James Prescott (1857), 'Some remarks on heat and the constitution of elastic fluids", *Phil. Mag.* 14, 211, reprinted in William Francis Magie (ed.), *A Source Book in Physics*, New York: McGraw-Hill, 1935, pp .

Keynes, John Maynard (1921), *A Treatise On Probability*, London: MacMillan.

Keynes, John Maynard (1936), *The General Theory of Employment, Interest, and Money*, New York: Harcourt, 1964.

Knight, Frank (1921), *Risk, Uncertainty, and Profit*, New York: Kelley and Millman, 1957.

Mandelbrot, Benoit (1963), 'The variation of certain speculative prices', *Journal of Business* 36: 394-419.

Mandelbrot, Benoit and Richard Hudson (2004), *The (Mis)behavior of Markets*, New York: Basic Books

Maxwell, James Clerk (1860), 'Illustrations of the dynamical theory of gases', originally published in *Phil. Mag.* 19, 19-32; 20, 21-37; reprinted in Stephen G Brush (ed.), *The Kinetic Theory of Gases*, London: Imperial College Press, 2003, pp. 148-71.

Mirowski, Philip (1989), *More Heat than Light: Economics as Social Physics, Physics as Nature's Economics*, Cambridge: Cambridge UP.

Runde, Jochen (2000), 'Shackle on Probability', in Peter E. Earl and Stephen F. Frowen (eds.), *Economics as an Art of Thought*, London: Routledge, pp .

Shackle, G.L.S (1939), 'Expectations and employment', *The Economic Journal* 49(195), 442-52.

Shackle, G.L.S (1940), 'The nature of the inducement to invest', *The Review of Economic Studies* 8 (1), 44-8.

Shackle, G.L.S (1949), 'A non-additive measure of uncertainty', *The Review of Economic Studies* 17 (1), 70-4.

Skidelsky, Robert (2003), *John Maynard Keynes: Economist, Philosopher, Statesman*, Oxford: Macmillan.

Stehle, Philip (1994), *Order, Chaos, Order*, Oxford: Oxford UP.

Taleb, Nassim Nicholas (2009), 'Error, robustness, and the fourth quadrant', *The International Journal of Forecasting* 25, 744-59.

Taleb, Nassim Nicholas and Avital Pilpel (2004), 'On the very unfortunate problem of the nonobservability of the probability distribution', unpublished manuscript, available at http://www.fooledbyrandomness.com/knowledge.pdf.

von Neumann, John and Oskar Morgenstern (1944), *Theory of Games and Economic Behaviour*, Princeton, NJ: Princeton UP.

Willes, Mark H. (1981), "'Rational Expectations' as a Counterrevolution', in Daniel Bell and Irving Kristol (eds.), *The Crisis in Economic Theory*, New York: Basic Books.

Nicholas Werle is an Affiliated Scholar at the Pembroke Center, Brown University, Providence (USA) (nickwerle@gmail.com)