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Pascal Stiefenhofer
Cristina Neesham

EDITURA ASE

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Pascal Stiefenhofer and Cristina Neesham

Abstract: Standard economic theory treats time as an exogenous, homogeneous parameter that indexes choice and coordination. We argue that this assumption is incompatible with a coherence-based account of rational agency. While objective time remains indispensable as a coordination device for social and economic organisation, it cannot account for the structure of individual decision-making. If rationality governs how action may proceed rather than which outcomes are selected, temporal structure cannot be presupposed independently of action but must arise from it.

We develop a formal framework in which agency is modelled as motion on a value-laden state space. An evaluative potential induces a local constraint on admissible directions of continuation, so that rational action is represented as coherence-preserving motion rather than optimisation. Within this structure, temporal order, direction, and rate emerge from evaluative progression along admissible trajectories. We establish that any temporal representation consistent with admissibility coincides, up to strictly monotone transformation, with this progression. Temporal structure is therefore not an independent modelling primitive but is fixed by the geometry of rational constraint.

This yields three implications. First, admissible continuation is generically non-unique, so rational divergence is structural rather than a deviation. Second, objective time organises coordination but does not determine the temporal structure of rational action, which is generated endogenously by admissible continuation. Third, uncertainty extends beyond outcomes to the temporal distance and rate at which they are realised, giving rise to temporal–metric uncertainty. A continuous-time economic–moral model shows how path dependence, disagreement, and heterogeneous temporal profiles arise directly from admissible structure, providing a generalisation of optimisation-based models in which time is not presupposed but generated by rational agency itself.

Keywords: economic temporality, utility theory, time asymmetry, time reversibility, emergent time, rational agency, arrow of time.

Introduction

In canonical economic models, time performs two distinct roles. It provides an ordering device for intertemporal comparison and coordination, and it serves as a constitutive presupposition of rational explanation, since preferences, beliefs, and plans are defined relative to a fixed temporal axis. Standard modelling practice collapses these roles. Temporal structure is treated as exogenous, linear, and analytically neutral (Newton 1833; Debreu 1959; Arrow 1971). Economic processes unfold within time, while time itself is excluded from explanation.

This treatment is not argued for. It is imposed as background structure. Time enters as a coordinate that indexes dated commodities or decision stages rather than as a feature of rational agency. In Debreu's *Theory of Value*, time is partitioned into analytically homogeneous intervals, within which all instants are indistinguishable for the purposes of analysis (Debreu 1959). A date becomes a position in a static commodity space. Temporal structure does not arise from agency. It is presupposed by it. The resulting representation is analytically powerful and structurally restrictive.

The restriction emerges once rational agency is understood as purposive activity. Action organises present conduct in relation to anticipated possibilities. Under genuine uncertainty, relevant futures cannot be exhaustively specified or assigned stable probabilities (Knight 1921; Keynes 1957). Temporal orientation is therefore internal to the structure of agency. A framework that treats time as independent of action cannot account for how action is directed toward an open and indeterminate future.

The limitation becomes decisive in dynamic analysis. Techniques such as dynamic programming and backward induction reconstruct present choice by fixing terminal conditions interpreted as future values (Bellman 1957; Stokey et al. 1989; Osborne and Rubinstein 1994). Reasoning proceeds over a temporal index that is invariant across admissible paths. Evaluation is therefore symmetric with respect to temporal direction at the level of formal reasoning. This symmetry stands in direct tension with the irreversibility of economic processes. The tension has long been recognised, but its source has not been identified (Ingrao and Israel 1990; Hart 1996). The temporal structure within which reasoning proceeds is treated as fixed rather than explained.

The problem is structural. When time is specified independently of action, temporal order, temporal distance, and temporal rate are fixed in advance of the processes they describe. Rational agency is defined relative to a framework that it neither generates nor transforms. Path dependence, irreversibility, and directional asymmetry are therefore excluded from temporal structure itself. The temporal parameter remains invariant while the processes it indexes may not.

Philosophical accounts of temporality reject this independence. In Kant, time is a condition for the possibility of experience rather than an external ordering (Kant 1999). In phenomenology, temporal structure is constituted through asymmetrical relations of retention and protention (Husserl 1991; Husserl 2006). In Heidegger, temporality is grounded in projection toward possibility (Heidegger 1962). In Bergson, duration is heterogeneous and irreversible (Bergson 1911). In process philosophy, temporal order emerges from becoming rather than from a pre-given sequence (Whitehead 1929). In each case, temporal structure is inseparable from the processes it organises.

The philosophy of economics advances a parallel critique of rationality. Sen's critique of the rational fool (Sen 1977; Sen 1999), theories of intention and planning (Bratman 1987; Aristotle 2011), and methodological analyses of modelling practice (Lawson 1997; Mäki 2001; Mäki 2018) reject the identification of rationality with optimisation. They emphasise coherence and intelligibility of action over time [1]. The temporal framework, however, remains unchanged. Rationality is revised while time remains fixed.

A gap follows. If rationality is not exhausted by optimisation and temporal structure is not independent of the processes it organises, then the role of time in rational explanation is mis-specified. Standard models presuppose a temporal structure that is incompatible with a coherence-based account of agency.

This paper establishes that incompatibility. It develops a framework in which rationality is formulated as a constraint on admissible continuation rather than as a rule for selecting optimal outcomes. The central question concerns how action may proceed. Within this framework, temporal structure is not presupposed. It is derived from the structure of admissible action. Temporal ordering and temporal passage arise from constraints governing coherent continuation rather than from an independently specified temporal parameter.

The remainder of the paper develops this argument.

Objective time, processual time, and the arrow of temporal structure

The standard representation of time in neoclassical economic theory imposes a fixed temporal structure on all admissible processes. Temporal order, distance, and rate are specified independently of the dynamics they describe and remain invariant across counterfactual evolutions of the system. This representation inherits its formal properties from classical mechanics, where time functions as a homogeneous background parameter that does not interact with the processes it orders (Newton 1833). In general equilibrium and intertemporal choice, this structure is embedded directly through dated commodities and temporally indexed decision nodes (Debreu 1959; Arrow 1971). Economic dynamics are therefore represented as variation within a predetermined temporal framework rather than as transformations of temporal structure itself.

This restriction becomes decisive when asymmetry is intrinsic to the process under consideration. In thermodynamic approaches to economic activity, irreversibility is not imposed but generated by the structure of transformation and dissipation (Georgescu-Roegen 1971). Temporal direction is therefore a consequence of process rather than a property of an external ordering. Within the standard economic representation, however, such asymmetry cannot be attributed to time. The temporal parameter remains invariant even when the processes it indexes are irreversible. As a result, the representation separates temporal structure from the very features that give rise to temporal direction. What is formally available is a description of asymmetric processes embedded in a symmetric temporal framework. The source of asymmetry is displaced from time to state dynamics.

Philosophical accounts of temporality reject this separation by locating temporal structure within the organisation of experience and action. In Kant, time is not a container but a form of intuition that conditions the possibility of synthesis. Temporal order is inseparable from the unification of representations in experience (Kant 1999). This introduces a dependence of temporal structure on the activity of synthesis itself. In Husserl, this dependence is sharpened through the structure of internal time-consciousness. Retention and protention generate a directed continuity in which the present is constituted through its relation to

what has just been and what is anticipated (Husserl 1991; Husserl 2006). Temporal asymmetry is therefore intrinsic to the structure of experience, not imposed externally. Heidegger radicalises this claim by grounding temporality in projection. The future is not a point on a timeline but a horizon of possibility that organises action (Heidegger 1962). Temporal direction emerges from the structure of anticipation itself. In Bergson, duration introduces heterogeneity into temporal structure. Time is not divisible into equivalent units but is constituted by qualitative change, so that temporal passage reflects the internal differentiation of process (Bergson 1911). Whitehead generalises this insight within a process ontology. Temporal order is the outcome of successive actualisations in which each event inherits and transforms the past, so that time is generated through becoming rather than presupposed as a container (Whitehead 1929). Each of these accounts identifies a distinct mechanism through which temporal structure depends on process, synthesis, anticipation, differentiation, or becoming.

This convergence has a precise implication. Temporal order and temporal direction cannot be specified independently of the structures that produce asymmetry. In the philosophy of physics, this claim is formalised by deriving the arrow of time from thermodynamic gradients, causal asymmetries, or informational structure (Reichenbach 1956; Price 1996; Rovelli 2015). Temporal direction is not primitive. It is induced by constraints on admissible processes. The temporal parameter becomes a derived object rather than a fundamental coordinate.

What is missing is a formal integration of this insight into a theory of rational agency. Economic theory retains an invariant temporal parameter even when modelling irreversible and path-dependent processes. Philosophical accounts establish the dependence of temporal structure on process but do not provide a formal representation of decision and action. The result is a division at the level of explanation. Temporal structure is either fixed without justification or derived without formal integration into models of agency.

This identifies a structural inconsistency. If rational agency is governed by constraints on admissible continuation, and if these constraints generate asymmetries in realised processes, then temporal structure cannot be specified independently of them. A representation that fixes temporal order, distance, and

rate in advance imposes invariance where the logic of action requires path dependence, irreversibility, and directional asymmetry. These features must then be displaced into state dynamics while the temporal framework itself remains unchanged. The result is a separation between the processes that generate temporal asymmetry and the structure used to represent them.

If rational agency is governed by constraints on how action may proceed, and if these constraints generate asymmetries in realised processes, then temporal structure cannot be specified independently of those constraints. A representation of time as a fixed parameter imposes invariance where the logic of action requires variation.

A reformulation is therefore required. Rather than presupposing a temporal framework within which action unfolds, the structure of time must be derived from the constraints governing admissible continuation. The relevant question is no longer how action is located in time, but how temporal structure is induced by the organisation of action itself.

Evaluative coherence and the structure of rational agency

This section introduces a coherence-based account of rational agency in which rationality is formulated as a constraint on admissible continuation rather than as a rule for selecting optimal outcomes. The question is not which states are chosen, but how action may proceed.

The term coherence is used here in a specific and restricted sense. The paper does not employ coherence as a theory of truth, epistemic justification, inferential consistency, or holistic belief integration in the philosophy-of-science tradition. Rather, coherence denotes the local preservation of evaluative intelligibility in the continuation of agency through time. An action is coherent insofar as its continuation does not locally negate the evaluative structure governing admissible motion. Coherence is therefore directional and processual rather than propositional. It concerns the admissibility of transition between states rather than logical consistency between beliefs or global optimisation over outcomes.

Standard economic theory adopts the opposite order of explanation. Rationality is defined over a temporally indexed space of outcomes, so that preferences, beliefs, and plans are organised relative to a fixed temporal parameter. Within this framework, rationality operates as a selection rule over a given set of states. The present approach rejects this ordering as primitive. Rationality is not defined by outcome ranking. It is defined by constraints governing continuation.

This perspective is consistent with coherence-based accounts of practical reason, which emphasise the intelligibility and stability of action over time rather than the maximisation of a scalar objective (Aristotle, 2011; Bratman, 1987). What is required, however, is a formal representation of such constraint. To that end, agency is modelled as motion in a structured space of action.

Let X be a smooth manifold representing the space of possible states. Evaluative commitments are represented by a smooth potential

$$U: X \to \mathbb{R}.$$

The role of U is local and directional. It does not induce a global ordering over states and is not interpreted as a quantity to be maximised. Instead, it determines which directions of motion preserve evaluative coherence at a given point.

At each state $x \in X$, the differential dU_x defines a set of admissible directions. Specifically, define the evaluative cone

$$C_U(x) \subset T_x X$$

as the set of tangent vectors along which the evaluative differential is weakly non-negative. The resulting cone field assigns to each state the directions in which action may proceed without locally undermining evaluative coherence, understood here as the preservation of admissible evaluative continuity in the progression of agency.

This construction defines rationality as a constraint on motion. Directions that violate evaluative coherence are excluded, but admissibility does not in general determine a unique continuation. Rational agency is therefore represented by trajectories whose velocities lie within the cone field at every point. Rationality governs admissible motion rather than outcome selection.

Evaluative admissibility and underdetermination

Underdetermination is a structural property of admissible continuation. By Corollary 1, a trajectory is admissible if and only if its instantaneous velocity lies in $C_U(x)$ at every point. Whenever $C_U(x)$ is non-singleton, multiple continuations remain admissible from the same state. Rational agency is therefore generically underdetermined, even when evaluative structure, feasibility, and information are held fixed.

This indeterminacy is not a limitation but a direct consequence of defining rationality as a constraint on continuation. Admissibility restricts motion without selecting it. A constraint on direction cannot, by itself, determine a unique trajectory. Determinacy arises only if additional structure is imposed.

The contrast with optimisation follows immediately. Optimisation defines rationality through global selection over outcomes. Admissibility defines rationality through local constraints on motion. What is determined is not a terminal state, but a set of permissible directions at each point. Rationality is therefore a property of continuation, not of outcome selection.

The relation between the two representations can be stated precisely.

Theorem 1 (Selection collapse and the emergence of optimisation). *Let $\mathcal{X}$ be a smooth manifold and let $U: \mathcal{X} \to \mathbb{R}$ be a smooth evaluative potential inducing the cone field*

$$\mathcal{C}_U(x) = \{v \in T_x\mathcal{X} \mid dU_x(v) \geq 0\}.$$

Let $\mathcal{A}(x) \subseteq T_x\mathcal{X}$ denote the feasible set of instantaneous motions at x.

Suppose there exists a single-valued selection operator

$$S(x) \in \mathcal{C}_U(x) \cap \mathcal{A}(x)$$

for all $x \in \mathcal{X}$.

Then admissible continuation is pointwise unique and the induced dynamics $\dot{x} = S(x)$ generate a single trajectory from each initial state.

If, in addition, S satisfies

$$dU_x\big(S(x)\big) \geq dU_x(v) \quad \text{for all } v \in \mathcal{C}_U(x) \cap \mathcal{A}(x),$$

then S coincides with a local maximising selection and the resulting dynamics recover the standard optimisation-based representation.

Theorem 1 identifies optimisation as a special case of admissible agency. When a single-valued selection is imposed, the admissible correspondence collapses to a vector field and the dynamics become determinate. Optimisation corresponds to a particular choice of selection that maximises the local evaluative increment.

Determinacy in standard models is therefore not a consequence of rationality itself. It is the result of imposing a selection that eliminates admissible alternatives. In the absence of such a restriction, admissible continuation remains non-unique. Rational multiplicity is thus a structural property of the framework.

Temporal structure from admissible motion

Once rational agency is characterised as admissible continuation, temporal structure must be derived from the properties of admissible motion.

By Theorem 3, evaluative admissibility implies that $U \circ \gamma$ is weakly increasing along any admissible trajectory γ. This monotonicity induces an intrinsic ordering along the trajectory. For any two points on γ, one precedes the other if and only if it corresponds to a lower level of evaluative progression. Temporal orientation is therefore defined by the ordering generated by admissible motion, rather than by reference to an external temporal parameter.

This ordering is purely structural. It requires neither optimisation nor a global temporal metric. It arises from the constraint that admissible motion preserves evaluative coherence. Temporal direction is thus internal to the structure of rational continuation.

Admissible motion also determines temporal passage. Let γ be an admissible trajectory. Define an agent-relative temporal coordinate τ along γ by evaluative progression. The local rate of temporal passage is then given by

$$\frac{d\tau}{dt} = dU_{\gamma(t)}(\dot{\gamma}(t)).$$

Temporal passage therefore tracks realised evaluative change along the trajectory. Where evaluative progression is greater, temporal passage is faster; where it vanishes, temporal passage stalls. Temporal rate is thus determined by the structure of admissible motion.

Because admissibility does not determine a unique trajectory, temporal structure is path dependent. Distinct admissible trajectories issuing from the same state induce distinct evaluative progressions and therefore distinct temporal coordinates. Temporal distance between states is not fixed independently of the path connecting them, but is defined by the realised trajectory.

This implies that agent-relative time is non-metric. It is not invariant across paths and cannot be reduced to a path-independent distance function on the state space. Temporal distance depends on how evaluative coherence is realised in motion.

Theorem 2 (Uniqueness of Temporal Representation). *Let $\gamma: I \to \mathcal{X}$ be an evaluatively admissible trajectory in the sense of Definition 1 and Corollary 1. Let $\tilde{\tau}: I \to \mathbb{R}$ be a temporal coordinate that is consistent with evaluative admissibility, in the sense that it is strictly increasing along every admissible trajectory. Then there exists a strictly increasing function $\phi: \mathbb{R} \to \mathbb{R}$ such that*

$$\tilde{\tau}(t) = \phi\left(U\big(\gamma(t)\big)\right) \quad \text{for all } t \in I.$$

Proof. Let $\gamma: I \to \mathcal{X}$ be evaluatively admissible. By Definition 1, admissibility requires that $\dot{\gamma}(t) \in \mathcal{C}_U\big(\gamma(t)\big)$ for all $t \in I$, which implies

$$dU_{\gamma(t)}\big(\dot{\gamma}(t)\big) \geq 0.$$

By Theorem 3, it follows that $U \circ \gamma: I \to \mathbb{R}$ is weakly increasing. By assumption, $\tilde{\tau}$ is consistent with admissibility, so it respects the ordering induced by admissible continuation. Since admissible trajectories are precisely those along which $U \circ \gamma$ is weakly increasing, $\tilde{\tau}$ must be strictly increasing on I. Now let $t_1, t_2 \in I$ satisfy

$$U\big(\gamma(t_1)\big) = U\big(\gamma(t_2)\big).$$

Because $U \circ \gamma$ is weakly increasing, it is constant on $[t_1, t_2]$. Suppose, for contradiction, that

$$\tilde{\tau}(t_1) \neq \tilde{\tau}(t_2).$$

Then $\tilde{\tau}$ would distinguish between two points that are indistinguishable under evaluative progression, contrary to consistency with admissibility. Hence

$$U\big(\gamma(t_1)\big) = U\big(\gamma(t_2)\big) \;\Rightarrow\; \tilde{\tau}(t_1) = \tilde{\tau}(t_2).$$

It follows that $\tilde{\tau}$ factors through $U \circ \gamma$. There therefore exists a function $\phi: \mathrm{Im}(U \circ \gamma) \to \mathbb{R}$ such that

$$\tilde{\tau}(t) = \phi\left(U\big(\gamma(t)\big)\right) \quad \text{for all } t \in I.$$

Finally, let $s_1, s_2 \in \mathrm{Im}(U \circ \gamma)$ with $s_1 < s_2$. Then there exist $t_1, t_2 \in I$ such that

$$U\big(\gamma(t_1)\big) = s_1, \qquad U\big(\gamma(t_2)\big) = s_2,$$

with $t_1 < t_2$ by monotonicity of $U \circ \gamma$. Since $\tilde{\tau}$ is strictly increasing,

$$\tilde{\tau}(t_1) < \tilde{\tau}(t_2),$$

hence

$$\phi(s_1) < \phi(s_2).$$

Thus ϕ is strictly increasing. Extending ϕ to $\mathbb{R}$ by standard monotone extension yields the result. QED.

This result expresses a structural limitation on temporal representation. Once admissible continuation is non-singleton, no path-independent temporal coordinate can be maintained without loss of coherence. A fixed temporal metric assigns identical temporal structure to trajectories that differ in their evaluative realisation. In doing so, it suppresses distinctions that are constitutive of admissible motion. The imposition of path-independence therefore forces a collapse of admissibility. Multiplicity is eliminated, and the space of coherent continuations is reduced to a single direction. Conversely, if admissibility is preserved, temporal structure cannot remain invariant. It must vary with the trajectory and therefore cannot be specified independently of it. Temporal representation becomes conditional on the structure of admissible motion.

Theorem 2 establishes this dependence in a formally exact sense. Attention is restricted to temporal coordinates that are admissibility-consistent. A temporal coordinate $\tilde{\tau}: I \to \mathbb{R}$ is admissibility-consistent if it satisfies two conditions. It is strictly monotone along every evaluatively admissible trajectory, and it is invariant on evaluatively indistinguishable states, in the sense that it is constant on level sets of $U \circ \gamma$. These conditions eliminate arbitrary parametrisations and restrict admissible representations to those that respect the ordering and equivalence structure induced by evaluative motion. Within this class, Theorem 2 yields a uniqueness result. Any admissibility-consistent temporal coordinate coincides, up to a strictly monotone transformation, with evaluative progression along admissible trajectories. There exists a strictly increasing function $\phi: \mathbb{R} \to \mathbb{R}$ such that

$$\tilde{\tau}(t) = \phi(U(\gamma(t)))\text{for all } t \in I.$$

This result exhausts the class of admissibility-consistent temporal representations. Once admissibility-consistency is imposed, temporal coordinates collapse to a single equivalence class under order-preserving transformation. No additional temporal structure can be introduced without violating the constraints governing admissible motion. Temporal form is therefore fixed by the evaluative geometry of continuation.

The implication is decisive at the level of rational explanation. Time does not function as an independent parameter indexing action. It arises from the ordering and magnitude of admissible progression. Any framework that posits a fixed temporal axis introduces invariance that is not supported by the logic of action. Such a parameter may retain an operational role in coordination or measurement, but it carries no autonomous explanatory content. The explanatory burden shifts entirely to the structure of admissible motion.

This conclusion provides a formal counterpart to a long-standing philosophical position. In Kant, temporal order is inseparable from the conditions of synthesis that render experience intelligible (Kant 1999). In Husserl, asymmetry is generated through the internal structure of retention and protention, so that temporal direction reflects the organisation of experience itself (Husserl 1991; Husserl 2006). In Reichenbach, temporal direction is derived from structural asymmetries rather than imposed as a primitive parameter (Reichenbach 1956).

Theorem 2 translates this line of reasoning into a formal result within a theory of rational agency. Temporal structure is not postulated but determined by admissible constraint.

The result also sharpens recent analyses of structural dependence in temporal representation. While existing work establishes that temporal form depends on underlying evaluative or conceptual structures (Stiefenhofer and Neesham 2026b; Stiefenhofer and Neesham 2026a), the present theorem shows that, under admissibility-consistency, this dependence is complete. There is no residual freedom in temporal representation once the structure of admissible continuation is fixed.

More generally, the argument instantiates a broader structural principle. Global organisation is induced by local admissibility constraints. In physical systems, this principle governs the emergence of flow architectures under irreversible conditions (Stiefenhofer and Deniz 2026). In the present framework, it governs temporal form. Temporal ordering and temporal rate are not imposed coordinates but necessary consequences of admissible motion. By Lemma 1 in Appendix B.4, every admissibility-consistent temporal coordinate factors through evaluative progression. Theorem 2 therefore does not impose temporal structure. It establishes that, once admissibility-consistency is required, temporal form is uniquely determined by the geometry of rational continuation.

Rationality, uncertainty, and philosophical positioning

Treating temporal structure as endogenous to evaluative admissibility requires a revision of uncertainty in rational explanation. In standard decision theory, uncertainty is defined relative to a fixed temporal framework. It concerns which state will obtain or which outcome will result from a given action at a given time, as in expected utility theory and its dynamic extensions (Savage 1954; Arrow 1971). Even approaches that relax full rationality, such as sparsity-based models, retain a fixed temporal metric and a predetermined ordering of decision stages (Gabaix 2014). Information-theoretic formulations extend the treatment of uncertainty while preserving this temporal invariance (Sims, 2003). In all

cases, uncertainty is defined over outcomes within an exogenously specified temporal structure.

This framework is incompatible with admissible continuation. When rationality constrains motion without selecting a unique trajectory, indeterminacy cannot be confined to states or outcomes. It arises at the level of trajectories. By construction, the admissible set $C_U(x)$ is generically non-singleton. Even when evaluative commitments, information, and feasibility constraints are fixed, multiple continuations remain admissible. Rational structure therefore does not determine a unique evolution. Uncertainty is intrinsic to action itself.

This yields a structural result. Whenever admissible continuation is non-unique, uncertainty over trajectories is unavoidable. This form of indeterminacy does not reduce to probabilistic uncertainty over outcomes or epistemic uncertainty about states. It arises from the form of rational constraint. Admissibility restricts direction without selecting it. A constraint on permissible motion cannot determine a unique path. Knight's analysis already recognises that economically relevant uncertainty need not admit probabilistic representation (Knight 1921). The present result is stronger. Even in the absence of outcome uncertainty, rational agency remains underdetermined whenever admissible continuation is non-unique.

Endogenising temporality introduces a further implication. Each admissible trajectory induces its own temporal ordering and rate through evaluative progression. Distinct trajectories therefore generate distinct temporal structures. Uncertainty extends beyond which outcomes occur to how they are temporally realised. Agents are uncertain not only about states, but about temporal distance and temporal passage. This defines temporal–metric uncertainty. By Theorem 5, temporal distance and temporal rate are functions of evaluative motion along a trajectory. Since admissible trajectories differ in evaluative progression, these quantities vary across rationally permissible paths.

This form of uncertainty cannot be represented within optimisation-based frameworks. Such frameworks require a fixed temporal metric and a determinate ordering of decision stages. Temporal distance and temporal rate are invariant across representations of the same transition. The possibility that

distinct admissible trajectories generate distinct temporal structures is therefore excluded by construction.

The implication is structural. Rational agency cannot be represented as outcome selection within a fixed temporal framework. Admissible continuation generates indeterminacy at the level of trajectories and induces temporal structure along them. Rationality, temporality, and uncertainty are jointly determined.

This result provides a formal counterpart to coherence-based accounts of rational agency. In Aristotelian accounts of practical reason, rationality concerns appropriate continuation in context rather than the maximisation of a scalar objective (Aristotle 2011). In Kantian accounts of purposiveness, unity of action over time is primary (Kant 1999). Phenomenological analyses locate temporal structure in anticipation and projection rather than in an external parameter (Husserl 1991; Heidegger 1962). The present framework identifies the formal mechanism. Temporal orientation and temporal passage arise from the asymmetry and path dependence of admissible continuation.

Treating time as exogenous is therefore not a neutral simplification. It excludes the dependence of temporal form on rational agency and misrepresents the structure of purposive action.

Temporal orientation and evaluative progression

This section instantiates the general framework in a fully specified economic–moral choice setting. The construction is exact. It realises the implications of evaluative admissibility within a deterministic and low-dimensional environment whose behaviour is fully determined by its primitives.

Formally, the construction specialises the framework of Appendices A–B. The feasible set $F \subset \mathbb{R}^2$ is defined by (11), the evaluative potential U by (12), and admissible motion by (13), with full specification given in Appendix C. The environment contains no stochastic elements, informational asymmetries, or preference variation. All behavioural variation arises from the geometry of admissible continuation.

An agent's situation is represented as motion in evaluative space $(x, m) \in F$. The potential $U: X \to \mathbb{R}$encodes evaluative structure locally. It does not induce a global ordering over outcomes. Instead, it determines which infinitesimal directions preserve evaluative coherence at each state.

At each point $x \in X$, admissible directions of continuation are given by the cone field

$$C_U(x) = \{v \in T_x X \mid dU_x(v) \geq 0\},$$

as defined in Definition 1 and instantiated in (13). Admissible trajectories γ are curves whose velocity lies in $C_U(x)$ at every point, as stated in Corollary 1.

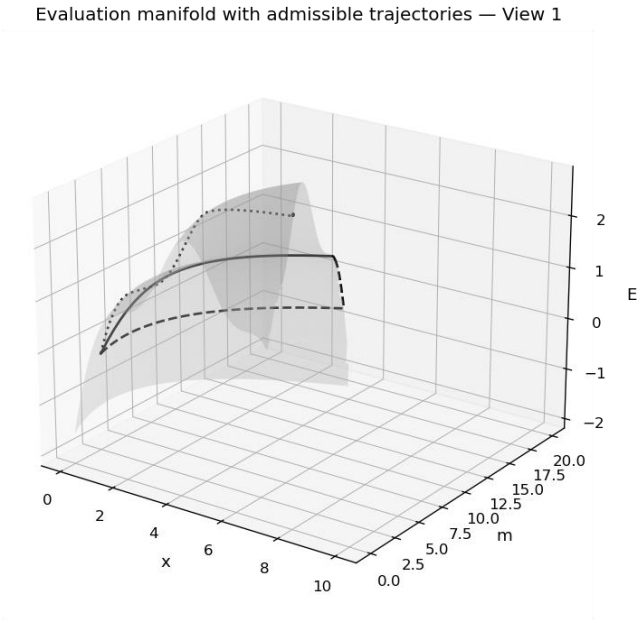
The admissible set $C_U(x) \cap A(x)$, defined in Appendix C, is generically non-singleton. Local continuation is therefore underdetermined. From any given state, multiple admissible directions satisfy identical evaluative and feasibility constraints. Distinct trajectories arise through selection within this set, as specified by (14). This multiplicity is implied by the structure of admissibility itself.

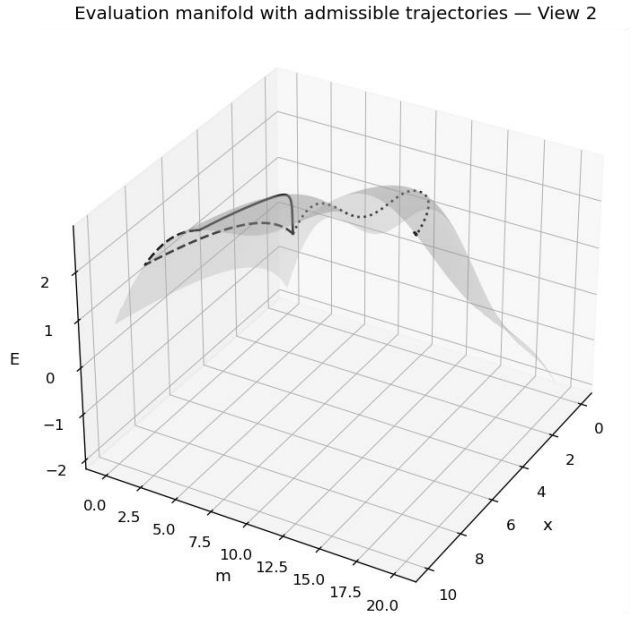
Figure 1 represents the resulting geometry. The surface is the evaluative potential U restricted to F, and the curves are admissible trajectories originating from a common initial condition. Each trajectory satisfies the same local admissibility condition $dU_x(v) \geq 0$ and identical feasibility constraints. The divergence of paths reflects the non-singleton structure of the admissible cone.

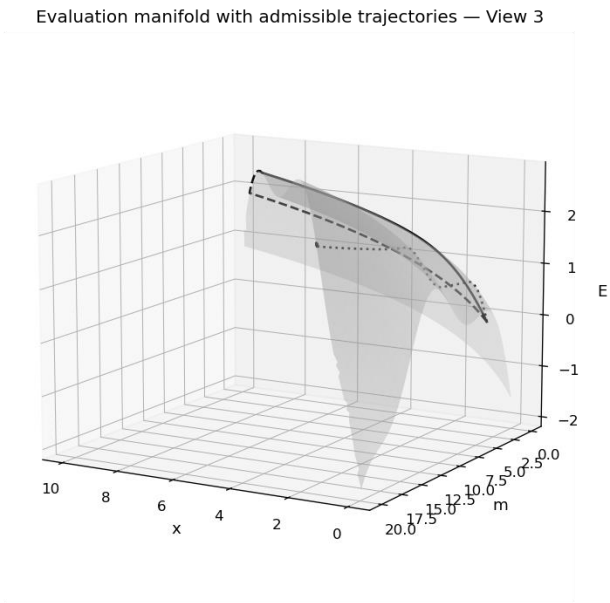
This divergence is structural. Identical primitives generate distinct admissible evolutions. No auxiliary variation is required. The multiplicity of trajectories is therefore a property of admissible continuation rather than of preferences, information, or noise.

Temporal structure is induced along each trajectory. As established in Appendix B, evaluative progression determines both temporal ordering and temporal rate. These quantities are defined relative to realised motion and therefore differ across admissible paths. Temporal structure is not imposed uniformly on the state space. It is generated along trajectories.

Figure 1 Evaluation manifold with admissible trajectories







Source: Authors' construction based on the mathematical model developed in this paper. Figure generated using Python 3.12 with NumPy, pandas, and Matplotlib.

Note: The surface represents the evaluative potential U over the feasible region F. Curves are admissible trajectories originating from a common initial condition and satisfying $dU_x(v) \geq 0$. Divergence reflects the non-singleton structure of the admissible cone, yielding multiple admissible continuations under identical primitives.

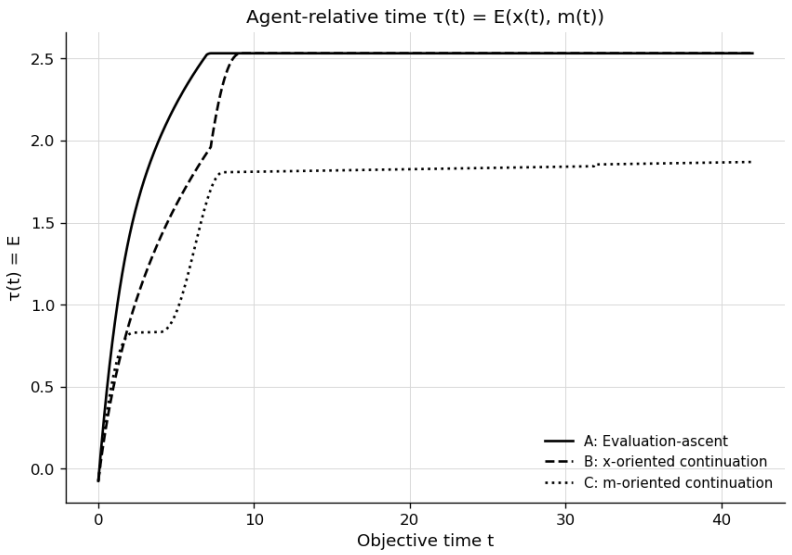
The construction isolates this mechanism. Path dependence, divergence, and heterogeneous temporal realisations arise directly from admissibility in a fully specified environment. The example therefore does not supplement the theory. It realises it.

Agent-relative temporal structure

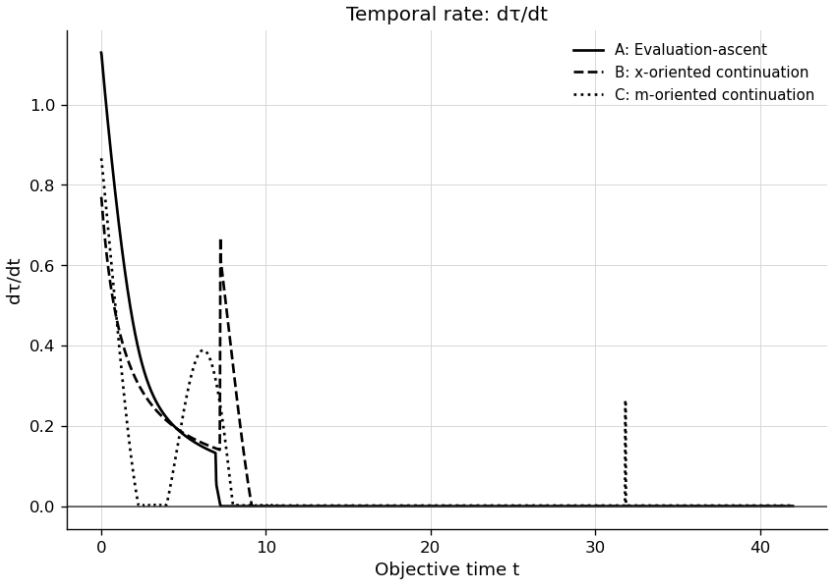
Temporal structure in the present framework is not imposed as an independent metric. It is induced by evaluative progression along admissible trajectories defined by (13). Figure 2 makes this dependence explicit by linking temporal ordering and temporal passage directly to evaluative motion under the potential (12).

Figure 2 Agent-relative temporal structure along evaluatively admissible trajectories

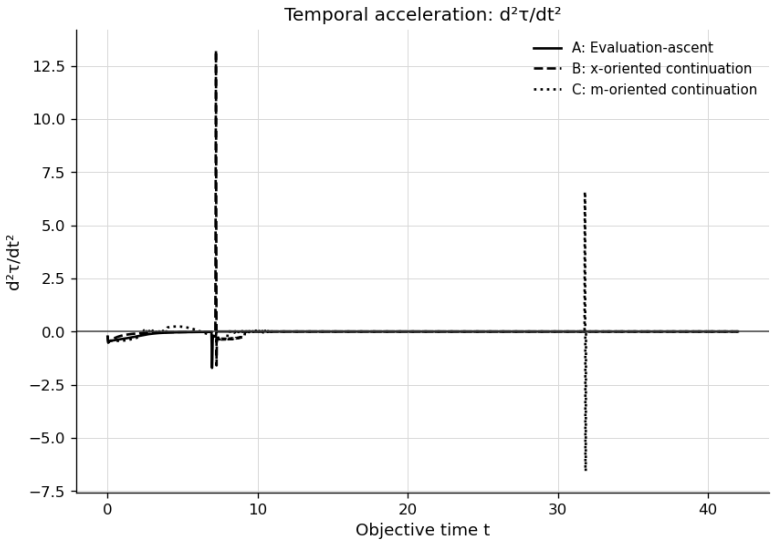
(a)



(b)



(c)



Note: Panel (a) displays evaluative progression and the induced temporal coordinate τ. Panel (b) shows the local temporal rate $\frac{d\tau}{dt}$ given by the evaluative derivative. Panel (c) illustrates endogenous variation in this rate along a trajectory, revealing acceleration and deceleration of temporal passage.

Panel (a) shows that temporal ordering is path dependent. For the same initial condition and evaluative structure (12), admissible trajectories generate distinct profiles of evaluative progression. Since τ is defined by accumulated evaluative change, temporal precedence is induced by the realised path rather than fixed independently of it. Panel (b) identifies the source of temporal passage. The rate of agent-relative time is given by $dU_{\gamma(t)}\big(\dot{\gamma}(t)\big)$, as established in Appendix B. Temporal passage therefore measures realised evaluative change. It increases with the magnitude of admissible progression and vanishes where evaluative motion is locally flat, independently of objective clock time. Panel (c) demonstrates that temporal passage is generically non-uniform. Variation in direction within the admissible cone $\mathcal{C}_U(x)$ alters the realised evaluative differential, producing endogenous acceleration and deceleration along a single trajectory.

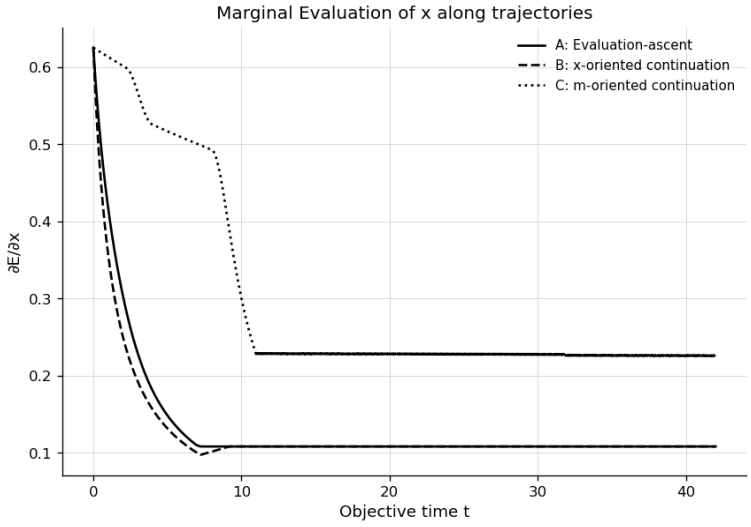
Together, Figure 2 identifies a precise structural property. Temporal ordering and temporal rate are functions of admissible motion and are therefore trajectory-dependent. No path-independent temporal metric is compatible with the admissibility structure defined in Appendix C. Temporal structure is thus induced, not imposed.

Marginal evaluation and realised change

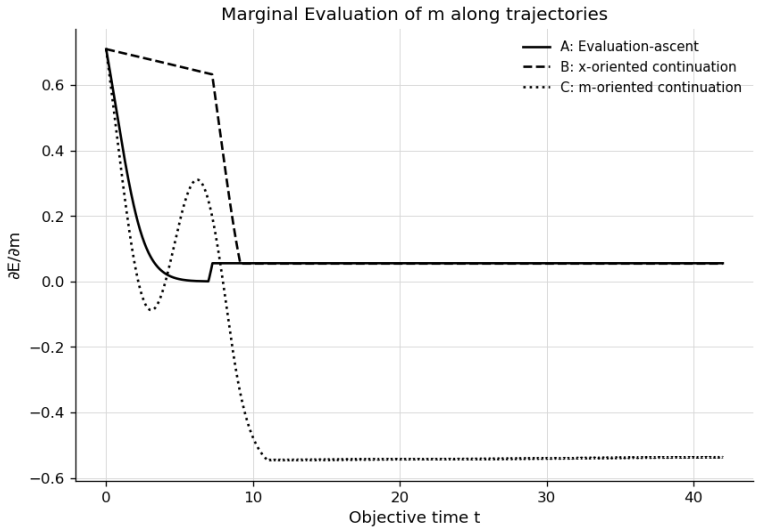
Figure 3 isolates a distinction that is decisive for the critique of optimisation-based reasoning: the difference between marginal evaluation and realised evaluative change along admissible trajectories defined by (13).

Figure 3 Marginal evaluation and realised evaluative change

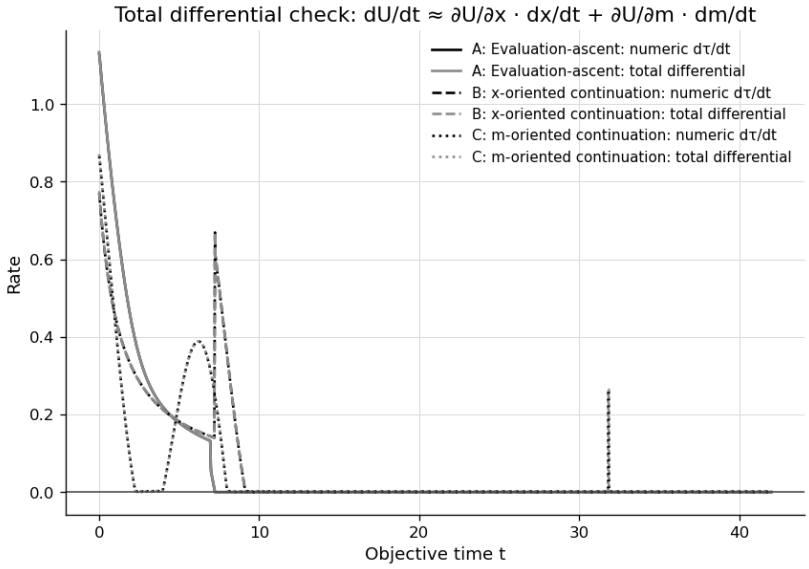
(a)



(b)



(c)



Note: Panels (a) and (b) display the components of the differential dU_x associated with (12). Panel (c) shows realised evaluative change $dU_x\big(\dot{\gamma}(t)\big)$ along admissible trajectories.

Panels (a) and (b) represent marginal evaluative structure. They correspond to the components of the differential dU_x of the potential (12) and therefore depend only on position in the evaluative space. At a given state, these quantities are identical across all trajectories. Agents may thus exhibit complete agreement about local evaluative structure. Panel (c) shows that such agreement does not determine realised change. Evaluative progression is given by the total differential $dU_x(\dot{\gamma}(t))$, which depends on the direction of motion selected within the admissible cone $\mathcal{C}_U(x)$. Although dU_x is fixed, admissibility permits multiple directions satisfying (13), and these directions generate distinct realised increments. This identifies a structural limitation of optimisation-based reasoning. In such frameworks, marginal evaluation is taken to determine a unique direction of adjustment. Here it performs a different role. It restricts admissible motion without selecting it. Rationality is therefore defined at the level of continuation rather than marginal maximisation.

A direct implication is that agreement about evaluation does not imply convergence of action. Distinct admissible trajectories can realise evaluative structure in different ways while remaining equally coherent. Divergence is therefore not a failure of rationality. It is implied by the admissibility structure itself. This distinction underpins the broader results of the paper. Because realised evaluative change depends on admissible direction, temporal structure and path dependence inherit this variation. The separation between marginal evaluation and realised change thus provides the mechanism through which rational disagreement, moral compromise, and agent-relative temporality arise within a fully specified environment.

Structural implications for rationality and time

The example identifies a structural source of behavioural diversity. With feasibility (11), evaluative structure (12), and admissibility (13) fixed, multiple admissible trajectories arise. This multiplicity is generated by the non-singleton geometry of $\mathcal{C}_U(x)$ and persists under identical primitives. As illustrated in Figure 4 (Appendix), trajectories issuing from a common initial condition diverge despite sharing the same evaluative potential and feasibility constraints. Path dependence, compromise, disagreement, and heterogeneous temporal

realisations therefore follow directly from admissible constraint rather than from variation in preferences, information, or environment.

This shifts the explanatory burden. Divergence does not require auxiliary mechanisms. In optimisation-based models, behavioural variation is typically attributed to preference instability, informational asymmetry, bounded rationality, or stochastic disturbance. Here, none of these are operative. Distinct admissible evolutions emerge from the same state. Disagreement and compromise are therefore internal to rational structure, not deviations from it.

A second implication concerns the architecture of rational explanation. Evaluative structure, feasibility, and selection are no longer collapsed into a single optimisation problem. Evaluative structure, given by (12), defines admissible directions. Feasibility, given by (11), delimits the attainable region. Selection, governed by (14), operates within the admissible set. This decomposition is necessary to represent normative constraint without reducing it to outcome ranking and to capture trade-offs as intrinsic to rational continuation.

A third implication concerns temporality. Temporal ordering and temporal rate are induced by evaluative progression along admissible trajectories, as formalised in Appendix B and illustrated in Figures 2 and 4. Because admissible trajectories differ, the induced temporal structures differ as well. Temporal representation is therefore constrained by admissibility: any admissible temporal coordinate must coincide, up to monotone transformation, with evaluative progression $dU_{\gamma(t)}(\dot{\gamma}(t))$. Temporal structure is not freely specified; it is fixed by the geometry of admissible motion.

The conclusion is structural. Temporal structure is not a background parameter for rational explanation. It is generated by the unfolding of agency under evaluative constraint. Rationality, normativity, and temporality are jointly determined. Any framework that treats time as exogenous while specifying rationality independently of admissibility introduces invariance that is not supported by the logic of action.

Conclusions

This paper establishes that rationality is not fundamentally a principle of outcome selection but a constraint on admissible continuation. Once agency is formulated in these terms, temporality can no longer function as an independent modelling primitive. Temporal order and passage arise from evaluative progression along admissible trajectories, while temporal rate is determined by realised evaluative change. Time, in the sense relevant for rational agency, is therefore endogenous to the unfolding of action itself.

This reversal of explanatory priority has direct consequences for uncertainty. Because admissible motion is generically non-unique, agency is underdetermined at the level of trajectories rather than merely outcomes. Uncertainty consequently extends to the temporal realisation of action, including the ordering, distance, and rate through which evaluative progression is experienced. Temporal–metric uncertainty is therefore intrinsic to admissible agency and cannot be represented within frameworks that presuppose a fixed temporal metric.

Optimisation appears within this framework only as a degenerate limiting case produced by collapsing admissible continuation to a single direction through an imposed selection rule. The determinacy of standard models therefore reflects an additional representational restriction rather than a constitutive feature of rationality itself.

The broader implication is that temporality, normativity, and agency cannot be treated as independent explanatory domains. Time does not provide a neutral background within which rational action unfolds, but emerges from the organisation of coherent continuation itself. Rational disagreement, path dependence, and heterogeneous temporal experience are therefore not deviations from rationality but consequences of the geometry governing admissible agency.

Appendix A. Formal structure of evaluative admissibility

This appendix formalises the notion of evaluative admissibility and shows how it induces intrinsic ordering and underdetermination of rational action without appeal to optimisation or an external temporal metric.

A.1 Evaluative space

Let $\mathcal{X} := \mathcal{X}^k \times \mathcal{X}^l$ denote an extended action space, where $\mathcal{X}^l \subseteq \mathbb{R}^l$ represents economic or experiential aspects of action and $\mathcal{X}^k \subseteq \mathbb{R}^k$ represents morally salient aspects (Stiefenhofer, 2025). Let $\mathcal{X} \subseteq \mathbb{R}^n$, with $n = k + l$, be a smooth manifold. Let

$$U: \mathcal{X} \to \mathbb{R} \tag{1}$$

be a smooth evaluative potential. No assumptions of concavity, monotonicity, or global representability by a complete preference ordering are imposed. The role of U is local and directional rather than global and aggregative.

A.2 Evaluative cone field

Let $T\mathcal{X}$ denote the tangent bundle of $\mathcal{X}$. For each $x \in \mathcal{X}$, the differential

$$dU_x: T_x\mathcal{X} \to \mathbb{R} \tag{2}$$

is a continuous linear functional.

Definition 1 (Evaluative Cone Field). *Let $\mathcal{X}$ be a smooth manifold and let $U: \mathcal{X} \to \mathbb{R}$ be a smooth evaluative potential. For each $x \in \mathcal{X}$, define the evaluative cone at x by*

$$\mathcal{C}_U(x) := \{v \in T_x\mathcal{X} \mid dU_x(v) \geq 0\}. \tag{3}$$

The collection

$$\mathcal{C}_U := \bigcup_{x \in \mathcal{X}} \mathcal{C}_U(x) \subset T\mathcal{X} \tag{4}$$

is called the evaluative cone field induced by U. Each fibre $\mathcal{C}_U(x)$ is a closed, convex cone.

Intuitively, $\mathcal{C}_U(x)$ specifies the directions in which agency may proceed without locally undermining evaluative coherence.

A.3 Admissible trajectories and intrinsic orientation

Let $\mathcal{X}$ be a smooth manifold and let $U: \mathcal{X} \to \mathbb{R}$ be continuously differentiable. An admissible direction at $x \in \mathcal{X}$ is given by the cone

$$\mathcal{C}_U(x) = \{v \in T_x\mathcal{X} \mid dU_x(v) \geq 0\}.$$

Let $\gamma: I \to \mathcal{X}$ be an absolutely continuous curve defined on an interval $I \subseteq \mathbb{R}$. The parameter $t \in I$ serves only as an external ordering device and carries no intrinsic temporal or normative content.

Definition 2 (Total Evaluative Differential). *Along a trajectory γ, the total evaluative differential is defined by*

$$\frac{d}{dt}(U \circ \gamma)(t) = dU_{\gamma(t)}\big(\dot{\gamma}(t)\big) \quad \textit{for almost every } t \in I. \tag{5}$$

Existence of admissible trajectories. To ensure that admissible motion is well defined, we impose standard regularity conditions on the cone field.

Assumption 1 (Regularity of the admissible cone field). *The correspondence $\mathcal{C}_U: \mathcal{X} \rightrightarrows T\mathcal{X}$ satisfies:*

i. $\mathcal{C}_U(x)$ *is non-empty, closed, and convex for all* $x \in \mathcal{X}$
ii. $\mathcal{C}_U$ *is upper hemicontinuous*
iii. $\mathcal{C}_U(x)$ *is locally bounded*

Under Assumption 1, standard results for differential inclusions ensure that for every $x_0 \in \mathcal{X}$ there exists an absolutely continuous trajectory $\gamma: I \to \mathcal{X}$ satisfying

$$\dot{\gamma}(t) \in \mathcal{C}_U\big(\gamma(t)\big) \quad \textit{for almost every } t \in I. \tag{6}$$

Evaluatively admissible motion is therefore well defined.

Theorem 3 (Local Evaluative Admissibility and Induced Orientation). *Let $\gamma: I \to \mathcal{X}$ be an absolutely continuous curve. If*

$$dU_{\gamma(t)}\big(\dot{\gamma}(t)\big) \geq 0 \quad \textit{for almost every } t \in I, \tag{7}$$

then $\dot{\gamma}(t) \in \mathcal{C}_U\big(\gamma(t)\big)$ *for almost every* $t \in I$*, and the scalar field* $U \circ \gamma$ *induces a canonical intrinsic ordering on the image* $\gamma(I)$.

Proof. By Definition 2,

$$\frac{d}{dt}(U \circ \gamma)(t) = dU_{\gamma(t)}\big(\dot{\gamma}(t)\big) \quad \textit{for almost every } t.$$

Condition (7) implies $\dot{\gamma}(t) \in \mathcal{C}_U\big(\gamma(t)\big)$ by definition of the cone field. Hence $U \circ \gamma$ is weakly increasing along the trajectory. This monotonicity induces a total preorder on $\gamma(I)$ defined by

$$\gamma(t_1) \preccurlyeq \gamma(t_2) \quad \Leftrightarrow \quad (U \circ \gamma)(t_1) \leq (U \circ \gamma)(t_2),$$

which is intrinsic in the sense that it depends only on the evaluative structure and not on the external parameterisation. QED.

Corollary 1 (Admissible motion and local underdetermination). An absolutely continuous curve γ is evaluatively admissible if and only if

$$\dot{\gamma}(t) \in \mathcal{C}_U\big(\gamma(t)\big) \quad \textit{for almost every } t \in I.$$

If $\mathcal{C}_U\big(\gamma(t)\big)$ *is non-singleton at some* t*, admissible continuation is locally underdetermined.*

Proof. The equivalence follows directly from Definition 1 and Theorem 3. If $\mathcal{C}_U\big(\gamma(t)\big)$ is non-singleton, there exist distinct directions $v_1, v_2 \in \mathcal{C}_U\big(\gamma(t)\big)$. Both satisfy $dU_{\gamma(t)}(v_i) \geq 0$, so both preserve evaluative coherence to first order. Hence multiple distinct admissible continuations exist from $\gamma(t)$, establishing local underdetermination. QED.

Appendix B. Agent-relative temporal structure

This appendix shows how temporal ordering and temporal passage are induced by evaluative motion along admissible trajectories.

B.1 Agent-relative temporal coordinate

Definition 3 (Agent-Relative Temporal Coordinate). *Let* $\gamma: I \to \mathcal{X}$ *be an evaluatively admissible trajectory and fix* $t_0 \in I$. *Define the agent-relative temporal coordinate*

$$\tau(t) := U\big(\gamma(t)\big) - U\big(\gamma(t_0)\big). \tag{8}$$

The coordinate τ orders states by evaluative progression rather than by objective clock time.

B.2 Temporal rate and passage

Definition 4 (Agent-Relative Temporal Rate). *Along an evaluatively admissible trajectory* γ, *the local rate of agent-relative temporal passage is given by*

$$\frac{d\tau}{dt} = dU_{\gamma(t)}\big(\dot{\gamma}(t)\big). \tag{9}$$

Theorem 4 (Relation Between Evaluative Motion and Temporal Passage). *Let* $\gamma: I \to \mathcal{X}$ *be an evaluatively admissible trajectory, and let the agent-relative temporal coordinate be defined by*

$$\tau(t) := U\big(\gamma(t)\big) - U\big(\gamma(t_0)\big)$$

for some fixed $t_0 \in I$. *Then:*

i. τ *is weakly increasing along* γ.
ii. Temporal passage is path dependent whenever the evaluative cone field $\mathcal{C}_U$ *is non-singleton along admissible continuations.*
iii. Acceleration and deceleration of agent-relative time correspond to variation in $dU_{\gamma(t)}\big(\dot{\gamma}(t)\big)$ *along the trajectory.*

Proof. For item (1), by Definition 3,

$$\tau(t) = U\big(\gamma(t)\big) - U\big(\gamma(t_0)\big).$$

Differentiating with respect to t and applying the chain rule gives

$$\frac{d\tau}{dt}(t) = dU_{\gamma(t)}\big(\dot{\gamma}(t)\big).$$

Since γ is evaluatively admissible, Definition 1 implies that

$$dU_{\gamma(t)}\big(\dot{\gamma}(t)\big) \geq 0 \qquad \text{for all } t \in I.$$

Hence

$$\frac{d\tau}{dt}(t) \geq 0 \qquad \text{for all } t \in I,$$

so τ is weakly increasing along γ. For item (2), suppose that at some state $x \in \mathcal{X}$ the cone $\mathcal{C}_U(x)$ is non-singleton. Then there exist distinct admissible directions $v_1, v_2 \in \mathcal{C}_U(x)$ with

$$dU_x(v_1) \geq 0 \qquad \text{and} \qquad dU_x(v_2) \geq 0.$$

By Corollary 1, each such direction can serve as the initial velocity of an evaluatively admissible continuation through x. Let γ_1 and γ_2 be two admissible trajectories issuing from the same initial state x with distinct admissible velocities. Then their induced temporal rates satisfy

$$\frac{d\tau_1}{dt}(t) = dU_{\gamma_1(t)}\big(\dot{\gamma}_1(t)\big), \qquad \frac{d\tau_2}{dt}(t) = dU_{\gamma_2(t)}\big(\dot{\gamma}_2(t)\big).$$

Since the trajectories are distinct, the realised evaluative motion along them need not coincide. In particular, even when they connect the same initial state to the same or comparable regions of $\mathcal{X}$, the accumulated evaluative progression

$$\tau_i(t) = U\big(\gamma_i(t)\big) - U\big(\gamma_i(t_0)\big)$$

may differ across $i = 1,2$. Hence temporal passage depends on which admissible path is realised. It is therefore path dependent whenever admissible continuation is non-unique. For item (3), by Definition 4,

$$\frac{d\tau}{dt}(t) = dU_{\gamma(t)}\big(\dot{\gamma}(t)\big).$$

Thus the local rate of agent-relative temporal passage is exactly the evaluative derivative along the trajectory. If this quantity increases over an interval, then agent-relative time accelerates on that interval. If it decreases, then agent-relative time decelerates. Equivalently, whenever the second derivative exists,

$$\frac{d^2\tau}{dt^2}(t) = \frac{d}{dt}\Big(dU_{\gamma(t)}\big(\dot{\gamma}(t)\big)\Big),$$

so acceleration and deceleration of agent-relative time are determined by variation in the evaluative derivative along the path. QED.

B.3 Relation to objective time

Theorem 5 (Relation Between Agent-Relative and Objective Time). Let $\gamma: I \to \mathcal{X}$ be a smooth curve and define

$$\tau(t) := \int_{t_0}^{t} d\, U_{\gamma(s)}\big(\dot{\gamma}(s)\big)\, ds.$$

Then:

i. $\frac{d\tau}{dt} = dU_{\gamma(t)}\big(\dot{\gamma}(t)\big)$.

ii. *Along evaluatively admissible trajectories, τ is weakly increasing.*

iii. $\left|\frac{d\tau}{dt}\right|$ *determines the local rate of agent-relative temporal passage.*

iv. *Whenever* $dU_{\gamma(t)}\big(\dot{\gamma}(t)\big) \neq 0$,

$$dt = \frac{1}{dU_{\gamma(t)}\big(\dot{\gamma}(t)\big)}\, d\tau.$$

Proof. By definition,

$$\tau(t) = \int_{t_0}^{t} d\, U_{\gamma(s)}\big(\dot{\gamma}(s)\big)\, ds.$$

Item (1) follows by the Fundamental Theorem of Calculus:

$$\frac{d\tau}{dt}(t) = dU_{\gamma(t)}\big(\dot{\gamma}(t)\big).$$

Equivalently, using the chain rule and the total evaluative differential (Definition 2), we can write

$$\frac{d\tau}{dt}(t) = \frac{d}{dt}(U \circ \gamma)(t) = dU_{\gamma(t)}\big(\dot{\gamma}(t)\big).$$

For (2), if γ is evaluatively admissible then, by Definition 1 (and equivalently by Corollary 1), we have $\dot{\gamma}(t) \in \mathcal{C}_U\big(\gamma(t)\big)$ for all t, hence $dU_{\gamma(t)}\big(\dot{\gamma}(t)\big) \geq 0$. By (1) this implies $\frac{d\tau}{dt} \geq 0$, so τ is weakly increasing. (This is the same monotonicity

mechanism used in Theorem 3.) For (3), the agent-relative temporal rate is, by definition, $\frac{d\tau}{dt}$ (cf. Definition 4); therefore $\left|\frac{d\tau}{dt}\right|$ gives the local speed of agent-relative temporal passage along the parameter t. For (4), whenever $dU_{\gamma(t)}\big(\dot{\gamma}(t)\big) \neq 0$, (1) gives $d\tau = dU_{\gamma(t)}\big(\dot{\gamma}(t)\big)\,dt$, so rearranging yields

$$dt = \frac{1}{dU_{\gamma(t)}\big(\dot{\gamma}(t)\big)}\,d\tau.$$

QED.

B.4 Factorisation of admissibility-consistent temporal coordinates

This section establishes a structural property that underpins Theorem 2. It shows that any temporal coordinate consistent with evaluative admissibility must factor through evaluative progression. The result isolates the precise mechanism through which temporal representation is reduced to a single admissible form.

Lemma 1 (Factorisation of admissibility-consistent time). *Let $\gamma: I \to X$ be an evaluatively admissible trajectory, and let $\tilde{\tau}: I \to \mathbb{R}$ be admissibility-consistent. Then $\tilde{\tau}$ factors through $U \circ \gamma$, that is, there exists a function $\phi: Im(U \circ \gamma) \to \mathbb{R}$ such that*

$$\tilde{\tau}(t) = \phi\left(U\big(\gamma(t)\big)\right) \qquad \text{for all } t \in I.$$

Moreover, if $\tilde{\tau}$ is strictly increasing along γ, then ϕ is strictly increasing on $Im(U \circ \gamma)$.

Proof. Let $s(t) := (U \circ \gamma)(t)$. By Theorem 3, evaluative admissibility implies that s is weakly increasing on I. We first show that $\tilde{\tau}$ is constant on level sets of s. Let $t_1, t_2 \in I$ satisfy $s(t_1) = s(t_2)$. Without loss of generality, suppose $t_1 < t_2$. Since s is weakly increasing,

$$s(t_1) \leq s(t) \leq s(t_2) \quad \text{for all } t \in [t_1, t_2].$$

Hence $s(t) = s(t_1) = s(t_2)$ for all $t \in [t_1, t_2]$. Thus $\gamma([t_1, t_2])$ consists entirely of states that are evaluatively indistinguishable. By admissibility-consistency, $\tilde{\tau}$ must be invariant on such sets. Therefore,

$$\tilde{\tau}(t_1) = \tilde{\tau}(t_2).$$

It follows that

$$s(t_1) = s(t_2) \;\Rightarrow\; \tilde{\tau}(t_1) = \tilde{\tau}(t_2).$$

Define $\phi: \mathrm{Im}(U \circ \gamma) \to \mathbb{R}$ by

$$\phi(r) := \tilde{\tau}(t) \quad \text{for any } t \text{ such that } s(t) = r.$$

The preceding implication ensures that ϕ is well defined. By construction,

$$\tilde{\tau}(t) = \phi\big(s(t)\big) = \phi\Big(U\big(\gamma(t)\big)\Big),$$

establishing the factorisation in Lemma 1. If $\tilde{\tau}$ is strictly increasing along γ, let $r_1 < r_2$ in $\mathrm{Im}(U \circ \gamma)$ and choose t_1, t_2 such that $s(t_1) = r_1$ and $s(t_2) = r_2$. Monotonicity of s implies $t_1 < t_2$, hence

$$\phi(r_1) = \tilde{\tau}(t_1) < \tilde{\tau}(t_2) = \phi(r_2).$$

Thus ϕ is strictly increasing. QED.

Lemma 1 establishes the structural restriction implied by admissibility-consistency. Any admissible temporal coordinate must factor through evaluative progression along the trajectory. It is therefore determined by the mapping $t \mapsto U(\gamma(t))$. Theorem 2 strengthens this result. It shows that any admissibility-consistent temporal coordinate coincides with evaluative progression up to a strictly monotone transformation. Together, these results eliminate temporal representation as an independent degree of freedom. Once rationality is specified as admissible continuation, temporal structure is determined by the evaluative geometry of admissible motion.

B.5 Non-uniqueness and temporal divergence

This section establishes that local underdetermination of admissible motion generates path-level divergence and distinct temporal structures. The result

provides the formal basis for the claim that rational disagreement and temporal heterogeneity are structural properties of admissible agency.

Definition 5 (Agent-Relative Temporal Coordinate). *Let $\gamma: I \to X$ be an evaluatively admissible trajectory and fix $t_0 \in I$. Define the agent-relative temporal coordinate by*

$$\tau(t) := U\big(\gamma(t)\big) - U\big(\gamma(t_0)\big).$$

The coordinate τ orders states by evaluative progression rather than by objective clock time.

Theorem 6 (Non-uniqueness and temporal divergence). *Let $X \subseteq \mathbb{R}^n$ be compact and let $U: X \to \mathbb{R}$ be continuously differentiable. Let admissible motion be defined by the cone field*

$$C_U(x) := \{v \in T_x X \mid dU_x(v) \geq 0\}.$$

Suppose there exists an open set $\Omega \subseteq X$ such that $C_U(x)$ is non-singleton for all $x \in \Omega$. Then for every $x_0 \in \Omega$, there exist two admissible trajectories $\gamma_1, \gamma_2: [0, T] \to X$ with

$$\gamma_1(0) = \gamma_2(0) = x_0,$$

such that:

i. $\gamma_1 \neq \gamma_2$ *on a set of positive measure;*

ii. *the induced evaluative progressions differ, i.e.*

$$(U \circ \gamma_1)(t) \neq (U \circ \gamma_2)(t) \quad \text{for some } t \in (0, T];$$

iii. *the corresponding agent-relative temporal coordinates*

$$\tau_i(t) := U\big(\gamma_i(t)\big) - U(x_0), \quad i = 1,2,$$

are distinct and therefore define different temporal structures.

Proof. Fix $x_0 \in \Omega$. By assumption, $C_U(x_0)$ contains at least two distinct directions $v_1 \neq v_2$ such that

$$dU_{x_0}(v_1) \geq 0, \quad dU_{x_0}(v_2) \geq 0.$$

By continuity of dU, there exists a neighbourhood V of x_0 on which both directions remain admissible up to first order. Standard existence results for differential

inclusions ensure that there exist absolutely continuous trajectories γ_1, γ_2 satisfying

$$\dot{\gamma}_i(t) \in C_U\big(\gamma_i(t)\big) \quad \text{a.e.},$$

with initial conditions $\gamma_i(0) = x_0$ and initial velocities $\dot{\gamma}_1(0) = v_1$, $\dot{\gamma}_2(0) = v_2$. Since $v_1 \neq v_2$, the trajectories diverge locally, establishing (1). Now consider the induced evaluative progression. By definition,

$$\frac{d}{dt}(U \circ \gamma_i)(t) = dU_{\gamma_i(t)}\big(\dot{\gamma}_i(t)\big).$$

At $t = 0$, we have

$$dU_{x_0}(v_1) \neq dU_{x_0}(v_2)$$

for a generic choice of distinct admissible directions. Hence the instantaneous rates of evaluative change differ. By continuity, this difference persists on a nontrivial interval, implying

$$(U \circ \gamma_1)(t) \neq (U \circ \gamma_2)(t)$$

for some $t > 0$, establishing (2). Finally, by Definition 5, agent-relative time is given by evaluative progression. Hence

$$\tau_i(t) = U\big(\gamma_i(t)\big) - U(x_0).$$

Since the evaluative progressions differ, the induced temporal coordinates differ as well, establishing (3).

QED.

Theorem 6 shows that non-singleton admissibility does not merely generate multiple directions of continuation. It generates distinct realised trajectories with different evaluative progressions and therefore different temporal structures. Rational divergence and temporal heterogeneity are thus joint consequences of admissible constraint. Combined with Lemma 1, this yields a complete structural result. Temporal representation is uniquely determined along each admissible trajectory, but admissibility itself does not select a unique trajectory. The multiplicity of admissible motion therefore induces a multiplicity of temporal realisations. Temporal underdetermination is thus not residual. It is a direct implication of the geometry of admissible agency.

Appendix C. Formal structure of the illustrative example

This appendix provides the complete formal specification of the illustrative model used in the main body of the paper. The construction instantiates evaluative admissibility in a low-dimensional setting and generates path dependence, rational divergence, and endogenous temporal structure without optimisation or stochastic disturbance.

C.1 Feasible set

Let $X := \mathbb{R}^2_{\geq 0}$ denote the non-negative state space. An agent's state is given by $(x, m) \in X$, where $x \geq 0$ and $m \geq 0$ represent, respectively, economic (or experiential) and moral components of action. Let $p, q, w \in \mathbb{R}_{>0}$ denote strictly positive parameters, interpreted as prices and available resources. Feasibility is defined by the linear constraint

$$px + qm \leq w. \tag{10}$$

The feasible set is therefore

$$\mathcal{F} := \{(x, m) \in X \mid px + qm \leq w\}. \tag{11}$$

C.2 Evaluative potential

Evaluative structure is represented by a smooth potential

$$U: X \to \mathbb{R},$$

given by

$$U(x, m) = \log(x + 1) + \alpha \log(m + 1) + \beta \sin(\gamma m) - \delta(m - m^\star)^2, \tag{12}$$

where $\alpha, \beta, \gamma, \delta, m^\star \in \mathbb{R}_{>0}$ are fixed parameters. The function U is continuously differentiable on X and defines a local evaluative structure through its differential. It is not assumed to be concave or globally order-preserving. Non-convexity is intentional and reflects evaluative conflict rather than a technical artefact, ensuring that the induced admissible cone field is generically non-singleton.

C.3 Admissible motion

A trajectory $\gamma(t) = \big(x(t), m(t)\big)$ is evaluatively admissible if

$$\nabla U\big(x(t), m(t)\big) \cdot \big(\dot{x}(t), \dot{m}(t)\big) \geq 0 \quad \textit{and} \quad \big(x(t), m(t)\big) \in \mathcal{F}. \tag{13}$$

Condition (13) defines a cone field on $T\mathcal{F}$. At each state, admissible directions consist of those that do not locally decrease evaluative value.

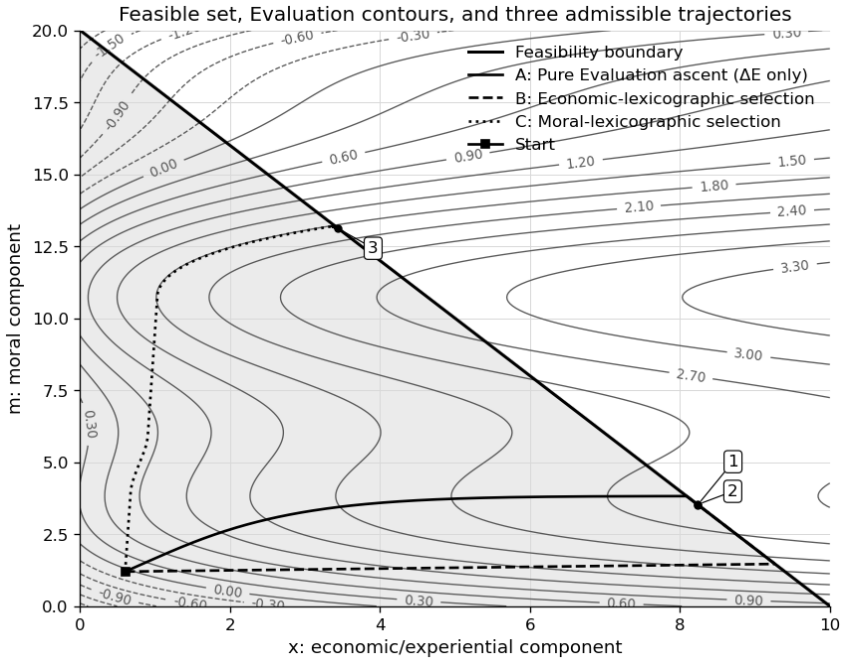
C.4 Selection of realised motion

Evaluative admissibility constrains continuation but does not determine it. At each state, the admissible set defines a correspondence of feasible directions rather than a unique velocity. To construct explicit trajectories for illustration, we introduce a selection rule that chooses a single direction from the admissible set. Specifically, let realised motion be selected by a policy mapping based on the score

$$S(v) = w_U \, dU(v) + w_x \, \dot{x} + w_m \, \dot{m}, \tag{14}$$

with fixed weights. Different weight vectors (w_U, w_x, w_m) induce distinct selections from the same admissible set and therefore generate distinct realised trajectories. This selection mechanism is not part of the definition of rationality. It serves only to produce concrete paths within the space of admissible continuations.

Figure 4 The shaded region denotes the feasible set defined by the budget constraint $px + qm \leq w$.



Note: Contour lines represent the evaluative potential $U(x, m)$. Three trajectories originate from a common initial condition and satisfy the admissibility constraint $\nabla U \cdot v \geq 0$ at every point, ensuring non-decreasing evaluative progression. The paths correspond to distinct admissible selection policies—evaluation-ascent, x-oriented continuation, and m-oriented continuation—yet share identical primitives (feasibility and evaluative structure). Their divergence illustrates the non-singleton geometry of the admissible cone: multiple rational continuations exist from the same state. Consequently, path dependence arises endogenously, and temporal structure—defined by evaluative progression along each trajectory—is trajectory-specific rather than imposed by an external time parameter.

Appendix D. Model reproducibility

Table 1 Reproducibility checklist for the two-state cone-admissible dynamics and plotting pipeline

Component	Specification / value
State variables	$x \geq 0$ (economic/experiential), $m \geq 0$ (moral)
Feasible set	$\mathcal{B} = \{(x,m) \in \mathbb{R}^2_{\geq 0} : px + qm \leq w\}$
Budget parameters	$p = 1.0,\ q = 0.5,\ w = 10.0$
Evaluation function	$U(x,m) = \ln(x+1) + 0.35\ln(m+1) + 0.45\sin(0.7m) - 0.025(m-8)^2$
Gradient	$\partial_x U = 1/(x+1);\quad \partial_m U = 0.35/(m+1) + 0.315\cos(0.7m) - 0.05(m-8)$
Objective time grid	$t \in [0,T]$ with $T = 42.0$ and uniform step $\Delta t = 0.02$
Initial condition	$(x_0, m_0) = (0.6, 1.2)$ (feasible; interior point)
Velocity magnitude	$\| v \| = v_{\max} = 1.2$ (fixed; direction varies)
Direction discretisation	$N_\theta = 1440$ directions, $\theta_k = 2\pi k / N_\theta$
Local admissibility	$\nabla U(x,m) \cdot v \geq 0$
One-step feasibility	$(x + \Delta t\, \dot{x},\ m + \Delta t\, \dot{m}) \in \mathcal{B}$
Discrete monotonicity	$U(t+\Delta t) \geq U(t) - \varepsilon,\ \varepsilon = 10^{-12}$
Policy score	$\text{score} = w_U\, \Delta U + w_x\, \dot{x} + w_m\, \dot{m}$
Policy A (evaluation-weighted)	$(w_U, w_x, w_m) = (3.0, 0.0, 0.0)$
Policy B (consumption-lexicographic)	$(w_U, w_x, w_m) = (1.0, 2.2, 0.05)$
Policy C (moral-lexicographic)	$(w_U, w_x, w_m) = (1.0, 0.05, 2.2)$
Subjective time	$\tau(t) = U\big(x(t), m(t)\big)$
Derived quantities	$\dot{x}, \dot{m}, \dot{\tau}, \ddot{\tau}$ via centred finite differences
Chain-rule check	$\dot{U} \approx U_x \dot{x} + U_m \dot{m}$ (numerical validation)
2D plotting grid	260×260 mesh over $x \in [0, w/p]$, $m \in [0, w/q]$
3D manifold grid	110×110 mesh; surface masked by feasibility
Manifold viewpoints	$(\text{elev}, \text{azim}) = (22, -55), (32, 30), (12, 120)$
Auxiliary vector field	Local best-admissible ascent: 31×31 grid; $N_\theta = 180$
Software	Python; NumPy; Pandas; Matplotlib; mpl_toolkits.mplot3d
Randomness	None (fully deterministic)

Note: The model evolves two nonnegative state variables, an economic/experiential coordinate x and a moral coordinate m, subject to a linear budget constraint. Dynamics proceed by selecting, at each step, a velocity direction from a discretised angular grid, enforcing local rational admissibility, one-step feasibility, and discrete monotonicity.

Among admissible directions, a policy-dependent score selects the realised velocity. All simulations are deterministic and fully specified by the parameters listed.

Endnotes

[1] The notion of coherence employed here is processual rather than epistemic. It concerns admissible continuity of evaluative agency and should not be conflated with coherence theories of truth, Bayesian coherence conditions, or inferential holism.

Conflict of Interest Statement

The authors declare that there is no conflict of interest.

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Pascal Stiefenhofer is a mathematician and economist in the Department of Economics, Newcastle University Business School (UK) (pascal.stiefenhofer@newcastle.ac.uk).

Cristina Neesham is Professor in the School of Business, Law and Entrepreneurship, Swinburne University of Technology (Australia), and Newcastle University Business School (UK) (cneesham@swin.edu.au).